A Simple Formula for Operational Risk Capital: A Proposal Based on the Similarity of Loss Severity Distributions Observed among 18 Japanese Banks*

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Abstract

We are proposing a simple formula that provides banks with an operational risk capital benchmark. The formula is built upon a statistical analysis of operational risk loss data from 18 major Japanese banks. It works well as a benchmark model and it is simple: the only parameter it uses to calculate operational risk capital is frequency of losses greater than or equal to a given severity. Yet it is risk sensitive, well-specified, and realistic.

Keywords: operational risk capital, Basel Capital Accord, benchmark model, loss data approach, extreme value theory, generalized Pareto distribution, power law

JEL Codes: G10, G20, G21, G28, G32

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1. Introduction

This paper analyzes operational risk loss data for 18 Japanese banks and proposes a simple benchmark formula for operational risk capital (ORC hereafter).

Under Basel II,[1] a bank that opts for the "Advanced Measurement Approach" (AMA) should develop its own ORC model and use this for regulatory purposes. This has led to the wider use of ORC models among banks worldwide, including Japanese banks. As a result, practices in operational risk measurement have made great strides.

However, there has been no approach for measuring ORC that deserves the name of an established standard; banks have a great deal of leeway when it comes to how they design and implement their models. In addition, improving measurement approaches through back testing is much more difficult than in other risk areas, all the more because Basel II requires "a soundness standard comparable to a one-year holding period and a 99.9th percentile confidence interval."[2]

Hence, the old problem of "the possibility that banks with similar risk profiles could hold different levels of capital under the AMA if they rely on substantially different modeling approaches and assumptions."[3] Furthermore, since the Basic Indicator Approach (BIA) of Basel II is already functioning as a *de facto* benchmark, it is possible that the AMA capitals for many banks have been greatly underestimated.[4]

Given this situation, operational risk measurement lacks the level of credibility of other risk areas, such as credit or market risk. This situation also makes it difficult to compare ORCs between banks.

While the standardization of modeling approaches seems premature,[5] it may be nevertheless highly desirable both for banks and regulators to have a benchmark that can be used to assess the reasonableness of any bank's ORC in order to reduce the disparities in ORC and prevent possible underestimation. Hence, what we are proposing is a benchmark for ORC, in the form of a simple formula that is much more effective at reflecting underlying risk than the BIA, which simply multiplies the gross income of a bank by 15% to obtain its ORC.

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1 The Basel Committee on Banking Supervision (2004)
2 Paragraph 657, The Basel Committee on Banking Supervision (2004)
3 The Basel Committee on Banking Supervision (2009a)
4 The Basel Committee on Banking Supervision (2009b) indicates that banks' AMA capitals vary to a much smaller extent than their loss frequency or severity figures; the AMA capital figures concentrate around a level, which is a little below the BIA capital (15% of the gross income).
5 "The flexibility provided in the AMA reflects the comparative stage of development of operational risk modeling, relative to the modeling of other risk types, and hence the need to allow banks to explore how best to obtain risk-sensitive estimates of operational risk exposure." (The Basel Committee on Banking Supervision (2009a))

We derive the formula by using a common severity distribution that we assume to be approximately applicable to every bank. The only parameter the formula uses to calculate ORC is frequency of losses greater than or equal to a given severity. It does not resort to Monte Carlo simulation.

It works well as a benchmark. By assuming a common severity distribution, the formula ignores the differences in the loss severity distributions between banks, but it simplifies the ORC calculation so as to be implementable by a wider range of banks. By using the annual frequency, it is still risk sensitive. It gives reasonably realistic ORC.

A Generalized Pareto Distribution (GPD) is fitted to a dataset of losses that are lumped together from a sample group of 18 Japanese banks to estimate the common severity distribution. It is difficult to determine whether or to what extent it is reasonable to apply this distribution to all these banks without a sufficient amount of data, but tentatively we consider it appropriate to use this particular distribution for benchmark purposes.

Although we have derived our results from 18 Japanese banks' data, our findings and the formula we have obtained are likely to be valid for other Japanese banks as well and even for banks around the world, without major changes. We will be referring to some studies and loss data collection exercises that support similar observations that we have tested and used in developing our formula.

Of course, further analysis and accumulation of operational loss data are needed to determine the range of the applicability of our formula. We hope that this paper encourages those engaged in the field of operational risk management to continue their efforts to accumulate and analyze data, which will lead to even better benchmarks than our proposal.

The idea of the formula is straightforward and is not unique to this paper. As far as the authors know, Dutta and Perry (2006), and De Koker (2006) indicate a similar formula. Dutta and Perry (2006) used a similar formula to estimate the ORC of seven sample banks based on the assumption that large operational losses follow a power law. De Koker (2006) derived an ORC formula similar to ours based on his observation of the power-law behavior of operational losses. He likened his formula to an IRB for operational risk. His fundamental observation and formula are similar to ours.

The greatest difference between their studies and ours is that we derive a simple formula by assuming that a common loss severity distribution fits reasonably well for many banks. This simplification enables banks with fewer losses to obtain some idea of their ORC without having to estimate their own loss severity distributions themselves.

We make the assumption of a common loss severity distribution based on our observation that different banks have a similar pattern of loss severity distribution. This observation is shared by some literature, including De Koker (2006) ("severity estimates … appear to be fairly constant from firm to firm and across the business cycle") and de Fontnouvelle et al (2005) ("we cannot reject the hypothesis that the distribution of losses is the same across large firms"). This paper examines this observation of similarity in loss severity distributions on the loss data from 18 Japanese banks.

There are many empirical studies on banks' operational losses that estimate the shape of the loss severity distribution and calculate their ORCs. Many of them provide more or less similar results to this paper. The studies are listed in Table 11 in Section 5.

The outline of the paper is as follows. We begin with a description of the data by observing that the loss severity distributions of operational losses of the 18 banks resemble each other and that they appear to be well approximated by a common loss severity distribution (Section 2). In Section 3, we estimate the common loss severity distribution that can be applied as an approximation for the 18 individual Japanese banks. We do this by fitting a GPD to a consolidated dataset of all the losses from the 18 sample banks. We then conduct goodness-of-fit tests of this common loss severity distribution to each individual bank's loss dataset. In Section 4, we introduce a simple formula for ORC, using the common loss severity distribution. We also evaluate how well the formula performs. Section 5 looks at other studies that indicate the applicability of the common loss severity distribution to other banks in Japan and abroad. Finally in Section 6 we conclude by discussing challenges in implementing the formula.

2. Data (loss data of 18 Japanese banks)

2.1 Outline of the data

We analyze a dataset reported by 18 Japanese banks as a part of the loss data collection exercise conducted by the Basel Committee on Banking Supervision in 2008 (LDCE2008 hereafter).[6] The main features of the dataset relevant to our analysis (most of them are common to the description in the LDCE2008 report) are:

- The analyzed dataset was submitted by 18 Japanese banks (including consolidated

[6] The report for the exercise is entitled, "Results from the 2008 Loss Data Collection Exercise for Operational Risk" (The Basel Committee on Banking Supervision (2009b)).

subsidiaries, such as securities companies), of which seven banks were targeting or had already implemented the AMA. All the other 11 banks had implemented the Standardized Approach (TSA).

- Data on 324,623 losses were submitted by the 18 banks. Of these, 2,502 losses were greater than or equal to 20K Euros, and the total amount of the losses was about 150 billion Yen (about 950 million Euros).[7]
- Institutions that participated in the LDCE2008 were asked to submit a minimum of three years of loss data. The period for submitted data varied from bank to bank, but on average, the 18 banks submitted a little over four years of loss data.
- Most of the 18 Japanese banks submitted data up to December 31, 2007, or March 31, 2008.
- Information requested for each loss event included:
  - ✓ Three dates related to the loss (date of occurrence, date of impact, and date of discovery);
  - ✓ The Basel business line and event type;
  - ✓ Gross loss amount, recoveries, and the amount of any insurance recoveries.
- The loss severity used in this paper is the gross loss amount after all recoveries (except insurance), i.e., gross loss amount − (all recoveries − insurance recoveries).
- "The aggregated dataset" for the Japanese banks is used in the analysis.[8]
- No data are excluded because of their age. As a result, five years of losses are used for a bank that submitted losses for five years and three years of losses for a bank that submitted losses for three years.
- The threshold of data varies depending on the analysis. When the threshold is set at a high level (e.g., 100K Yen, about 634 Euros), less than 18 banks are analyzed, as some banks do not incur losses greater than that higher threshold.

2.2 Graphical presentation of the losses

Losses of individual banks (or groups of banks) are presented graphically in a manner that respects their anonymity. In this paper we call these graphs double-log plots since both the *x* and *y* axes are in log terms. They are plotted as follows:

- All the losses greater than or equal to 10K Yen (about 63 Euros) are depicted;

[7] The exchange rate applied throughout this paper is for March 31, 2008 (1 Yen = 0.006336486 Euros). It is the same as the one used in LDCE2008. Please refer to The Basel Committee on Banking Supervision (2009b) for the details.

[8] The aggregate dataset was constructed by converting loss amounts into euro amounts and aggregating transactions with the same reference number into one event. The euro amounts were converted back to yen in our analysis. Note that the dataset mainly analyzed in the LDCE2008 was the stable dataset, a subset of the aggregate dataset containing data from a stable time period for each bank. Please refer to The Basel Committee on Banking Supervision (2009b) for details.

- The x-axis stands for the severity (Yen amount) of losses;
- The y-axis stands for: (the number of losses ≥ severity on the x-axis) / (total number of losses ≥ 10K Yen);*

  * $1-F(x)$; $F(x)$ being the cumulative distribution of the loss severity.
- Both axes are in log terms;
- For the purpose of anonymity, the scales of the x-axis and y-axis are changed from bank to bank, so that the maximum severity for each bank is located at roughly the same position;
- Losses from single banks (Figure 1) and losses from groups of different banks lumped together (Figures 2, 3) are depicted;
- No adjustments, such as weighting or scaling, to the frequency or severity of the losses are made when losses from different banks are graphed in one plot. In other words, losses from different banks are plotted without any adjustments, as if they occurred in different divisions or subsidiaries of a single bank;
- All the losses from any single bank or group of banks are depicted in one graph; i.e., graphs are drawn at an enterprise level, not at a more granular level, such as a business-line or event-type level;
- A continuous straight line with a slope of −1 is added for the purpose of comparing the graphs.

Figure 1 shows the loss severity distributions of the individual 18 banks.[9] To a certain extent, the curves of different banks resemble each other, with considerable variation in the tail region.

However, when losses from a set of banks are lumped together and plotted as if they were from one bigger bank, they exhibit a tendency to form an almost straight line with a slope of about −1.[10]

Figure 2 is an example. Losses from nine medium-sized Japanese banks (known as regional banks) are lumped together and plotted as if they were from one bigger bank. While the distributions of individual banks' losses look different (left), the consolidated data from the same nine banks form a nearly straight line with a slope of about −1 (right).

Figure 3 shows the same graph for a larger consolidated dataset that is taken from all the 18 sample banks. The curve forms a nearly straight line with a slope of about −1,

[9] More losses in the graph do not necessarily mean more annual losses, because the period of reported losses varies from bank to bank.

[10] The straight line with a negative slope means that the losses follow a power law and that a Pareto-type distribution can be fitted to them. The fact that the slope of the line is about −1 indicates the relationship in which, when a loss becomes 10-times as severe, the probability of such a loss occurring becomes 10 times less. This relationship is well in line with practical experience.

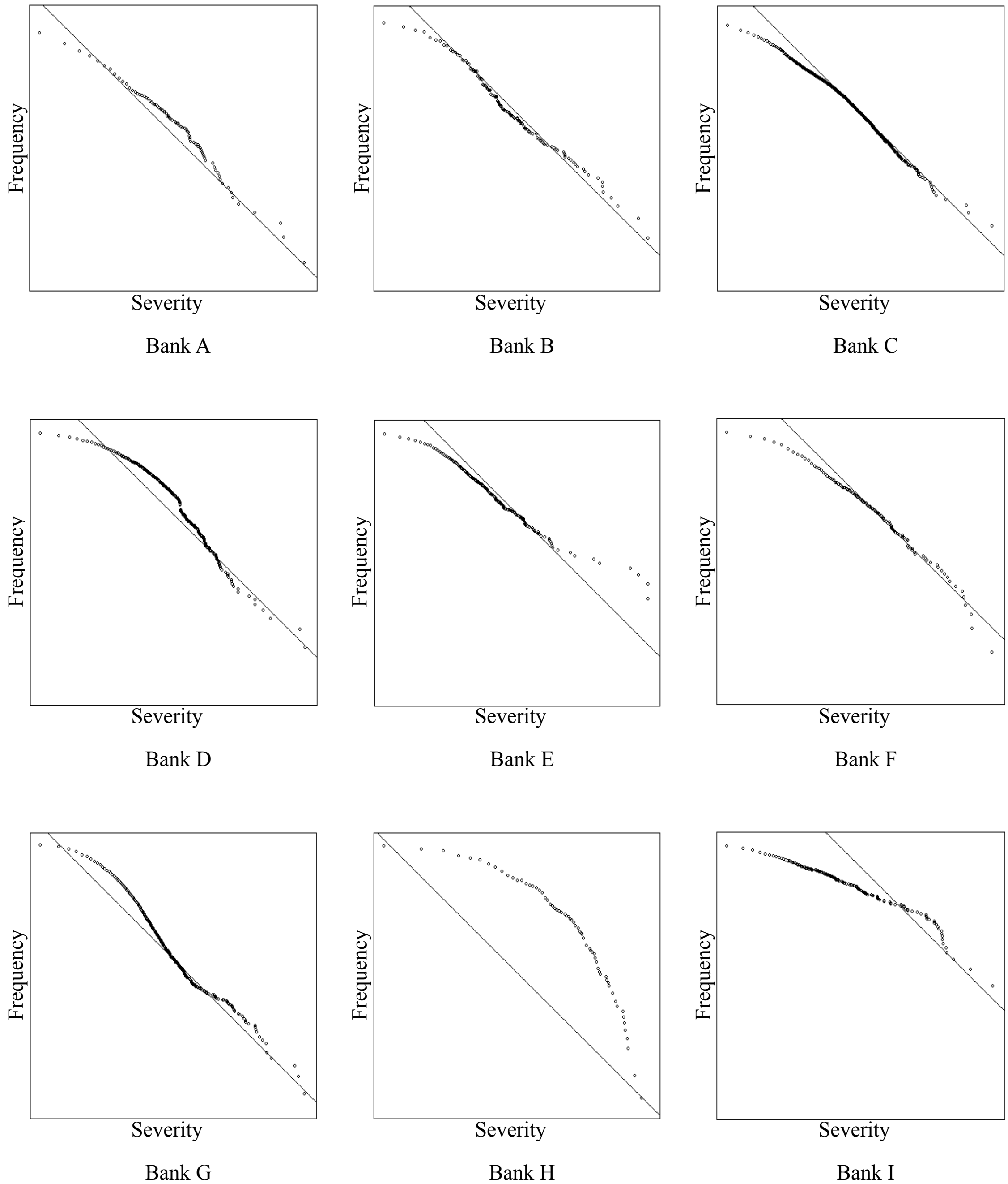
similar to the curve in Figure 2.

There are two ways a loss severity distribution based on the consolidated data can be interpreted. First, it can be thought of as a distribution for a hypothetical big bank that merges several banks together. That is, all the 18 banks are thought of as if they were subsidiaries that make up a single big bank, and it is the loss severity distribution of that bank which is examined.[11]

The other interpretation is to consider the dataset as if it had been obtained from many years of operation by a single bank. Since the 18 banks submitted an average of a little more than four years of losses, this distribution can be interpreted as the result of 70 to 80 repetitions of one year's operation of an average bank ($18 \times 4 = 72$).

If we adopt this latter interpretation, we might be justified in assuming that an individual bank's losses follow the loss severity distribution of Figure 3 and that they approach it when more loss data are accumulated over time.

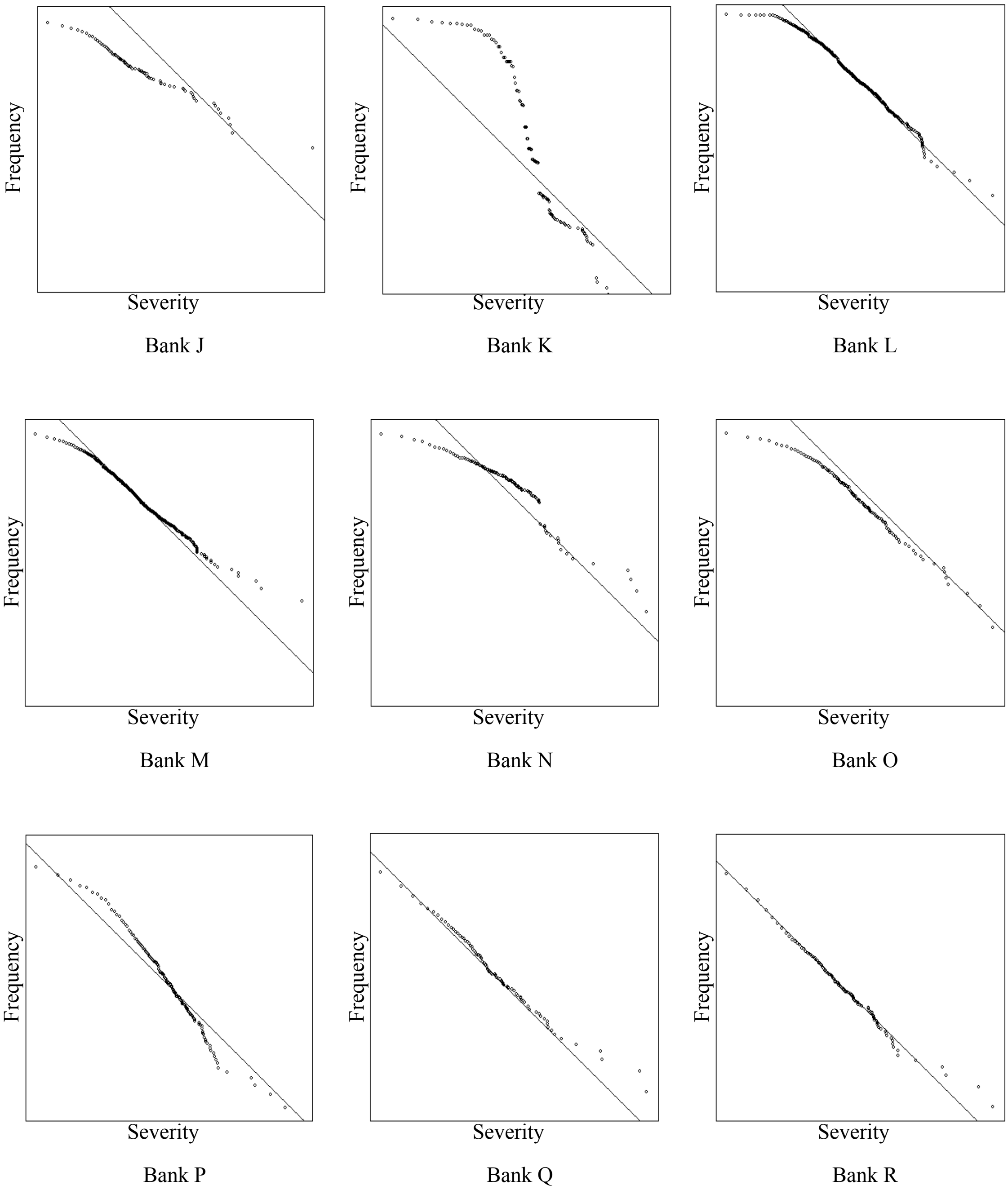
[11] All the losses from all the 18 banks need to be independent for this interpretation. Because of this, several losses in several banks that have the same root cause (e.g., an earthquake or disruption of a computer system shared by banks) need to be treated as a single loss.



$$\text{Frequency} = \frac{\text{number of losses} (\geq \text{severity on the } x\text{-axis})}{\text{total number of losses} \geq 10\text{KYen}}$$

Both severity (x-axis) and frequency (y-axis) are in log terms.

Figure 1 (continued below): Loss severity distributions for individual banks



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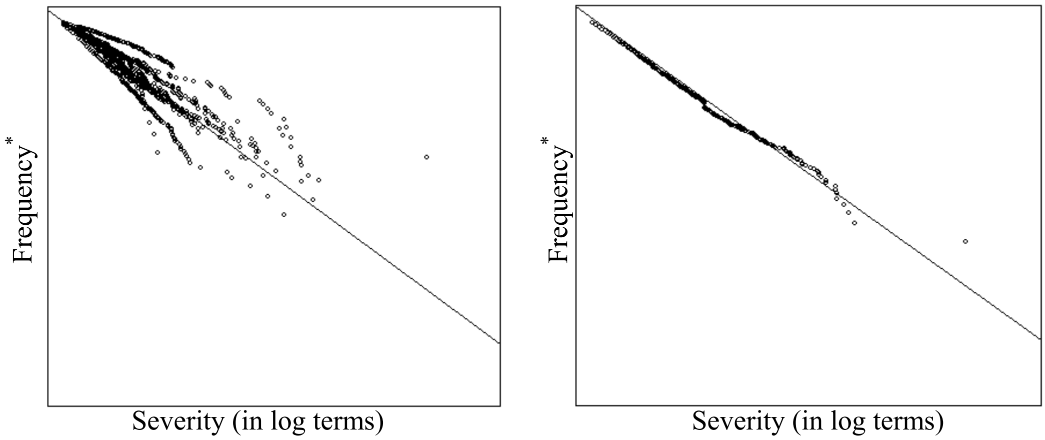
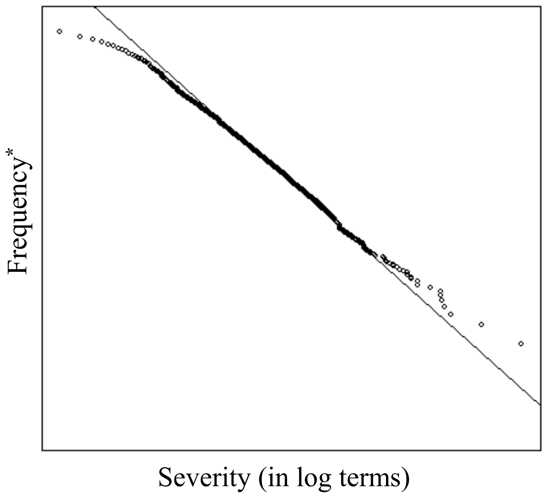


$$* \text{Frequency (in log terms)} = \frac{\text{number of losses} (\geq \text{severity on the } x\text{-axis})}{\text{total number of losses} \geq 10\text{KYen}}$$

Figure 2: Losses of nine medium-sized banks lumped together: individual datasets (left), consolidated dataset (right)



$$* \text{Frequency (in log terms)} = \frac{\text{number of losses} (\geq \text{severity on the } x\text{-axis})}{\text{total number of losses} \geq 10\text{KYen}}$$

Figure 3: Losses of all the 18 Japanese banks lumped together

3. Estimation of the common loss severity distribution

In the previous section we showed that by consolidating data, the loss severity distributions of separate banks appear to converge toward a Pareto-type distribution. In other words, we may be justified in assuming that individual banks' losses come from a Pareto-type common distribution ("common loss severity distribution" hereafter).

In this section, we first fit the Generalized Pareto distribution to the dataset of the 18 banks' losses lumped together to determine the common loss severity distribution.[12]

Then we examine the appropriateness of the assumption that individual banks' losses follow this common loss severity distribution. We do this because the fact that the consolidated dataset appears to follow the common loss severity distribution, e.g., the consolidated losses take a shape like that in Figure 3, does not necessarily mean that the individual banks' losses follow this distribution.[13]

We examine the appropriateness of the common loss severity distribution assumption through two kinds of statistical tests. First, we confirm in Appendix 1 that the null-hypothesis that the loss severity distribution is identical across 18 sample banks (the alternative: losses in at least one bank do not follow the same loss severity distribution) is not rejected. Second, we confirm in this section that the common loss severity distribution for the consolidated data fits reasonably well to the individual banks' data sets.

3.1 The i.i.d. assumption

We assume that the observations to which the distribution is fitted are the realizations of independent, identically distributed (i.i.d.) random variables.[14]

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[12] Nešlehová et al. (2006) points out the danger of data contamination, i.e., "there are observations within the sample which do not follow the same distribution as the rest of the data" and calls for special attention when contamination is suspected. The determination of the common loss severity distribution by fitting a GPD to the consolidated dataset of 18 sample banks as we did runs this same risk. However, we think our method is justified for the following reasons: (i) The individual banks' dataset can be assumed to follow the same distribution (see Footnote 14), (ii) The predictions of the maximum loss by the common loss severity distribution are conservative in half of the cases and less conservative in the other half the cases when compared with the actual losses for individual banks (see Table 2).

[13] For example, when the individual banks' loss datasets follow different log-normal distributions with different variance, the consolidated dataset from these banks can take a very similar shape to that in Figure 3. This also happens when some banks' losses follow power distributions and other banks' losses follow log-normal distributions. On the other hand, when several banks' losses individually follow the common loss severity distribution, then the consolidated dataset from these banks also follows that distribution.

[14] The i.i.d. assumption is justified by the same reasons provided in Moscadelli (2004), which analyzed a loss dataset collected by the Basel Committee on Banking Supervision. The study based the assumption of independence on the idea that any pooling exercise of banks' operational risk losses collected in a reasonably short time can mitigate the threat of dependence of the data. To a lesser

3.2 Units of measure

The unit of measure is at the enterprise level, not the business-line or event-type level. This is consistent with our purpose of deriving a simple ORC formula. It also allows a larger sample size. See Appendix 2 for estimations for individual business lines or event types.

3.3 Distributional assumptions

We fit the Generalized Pareto Distribution (GPD) to the excess value over a given threshold (see Section 3.4 below for the threshold) of losses greater than that threshold. In other words, we use the "Peaks over threshold" (POT) approach on the assumption that the threshold is sufficiently high.[15]

The GPD is expressed as follows (the case when $\xi = 0$ is omitted):

$$G_{\xi,\beta}(x) = 1 - \left(1 + \frac{\xi}{\beta} x\right)^{-\frac{1}{\xi}} \qquad \xi \neq 0, \qquad \text{where} \quad \beta > 0\text{, and} \quad x \geq 0 \quad \text{when} \quad \xi > 0$$

The parameters ξ and β are referred to as the shape and scale parameters, respectively.

3.4 Threshold for the GPD

GPD parameter estimates (Figure 4, Table 1) are highly sensitive to the thresholds above which the distribution is fitted to. Therefore choosing the threshold is crucial in fitting the GPD. However, there is no unique, established way to determine this value and it is usually the case in practical application that various qualitative and quantitative factors are taken into consideration.

In this paper, we are using mainly the set of parameters estimated with the threshold of 10 million Yen (about 63K Euros). This choice of the threshold is based on the following considerations.

- A higher threshold provides a better approximation of the data by the GPD, but a smaller number of samples. On the other hand, a lower threshold provides more observations and a smaller estimation variance but a bigger bias.
- We experimented with thresholds ranging from 1 million Yen (about 6K Euros) to

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degree, this situation holds true for our dataset as well, whose data are a little over four years of age on average. Moscadelli (2004) based the identically distributed data assumption on the consideration that his sample banks have characteristics that are not too dissimilar, because they were the ones that participated in the 2002 LDCE. This holds true, possibly to a higher degree, for our dataset, which is obtained from 18 middle-sized or large-sized Japanese banks.

[15] The Pickands-Balkema-de Haan Theorem essentially states that the "distribution of the losses in excess of a sufficiently high threshold can be approximated by the GPD."

over 1 billion Yen (about 6 million Euros) in 1 million Yen increments. ξ is stable for the thresholds between 1 million and about 30 million Yen (about 190K Euros). This holds true regardless of whether they are estimated by the maximum likelihood (ML) method or the probability weighted moments (PWM) method (Figure 4, Table 1).

- The mean excess plot is approximately linear for the whole area over 1 million Yen (6K Euros),[16] except for tail parts above 100 million Yen (about 634K Euros), where only a small number of large excesses are averaged (Figure 5). This suggests that the GPD threshold could be as small as 1 million Yen (about 6K Euros).
- A lower threshold used in the estimation of the GPD enlarges the applicability of the simple ORC formula we propose.[17]

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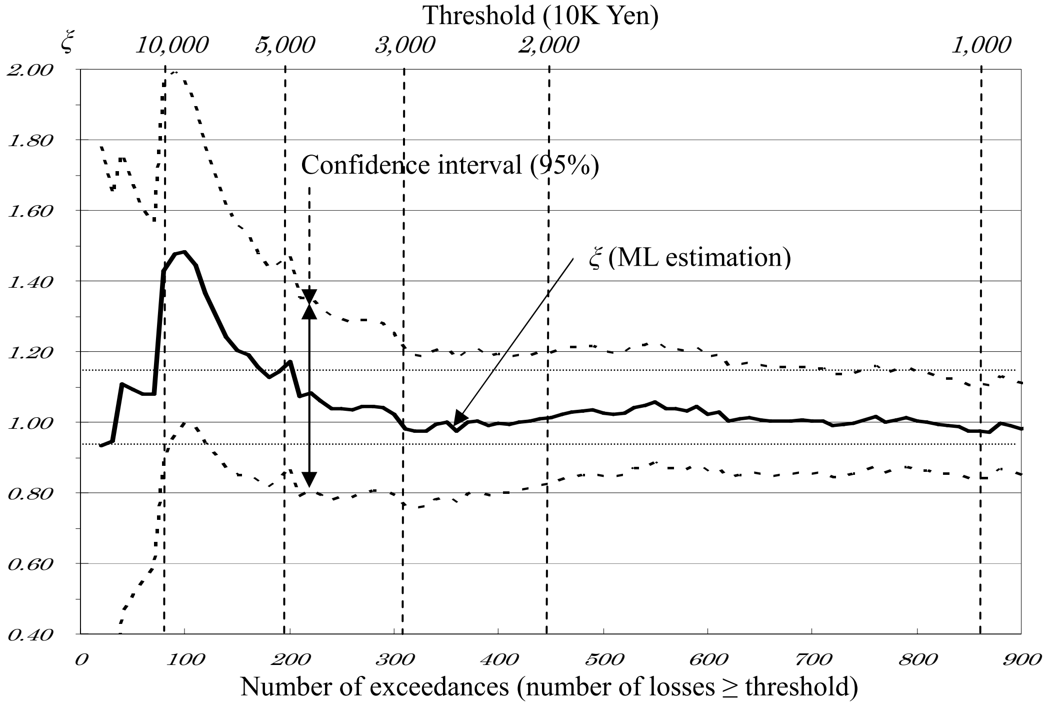
[16] The graph plots the following points of (x, y):
x-axis: various threshold levels u ($\geq$1 million Yen), y-axis: mean of the excess values (loss severity − threshold u) of all the exceedances (the losses greater than u). The biggest three losses are excluded from the graph.

[17] The ORC formula uses as input the annual frequency of losses that are greater than or equal to a given severity, which needs to be higher than or equal to the GPD estimation threshold. Therefore, a lower threshold for the GPD estimation allows us to use the frequency of smaller losses in the formula, thus widening the formula's applicability to banks with zero or few bigger losses.



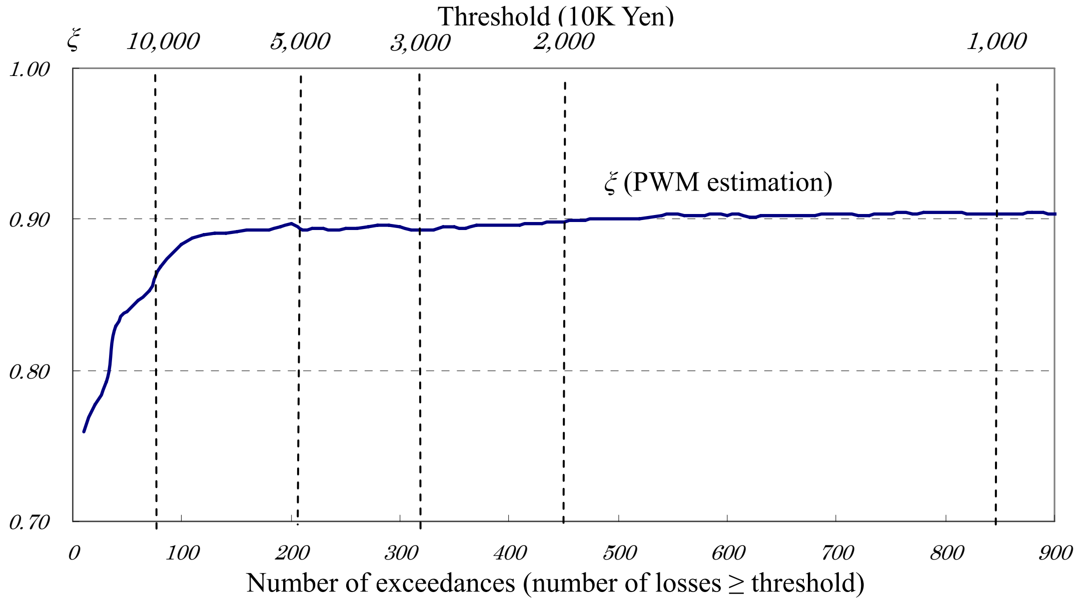


Figure 4: GPD Estimates of ξ for different numbers of exceedances and thresholds: estimates using ML method (top), estimates using PWM method (bottom)

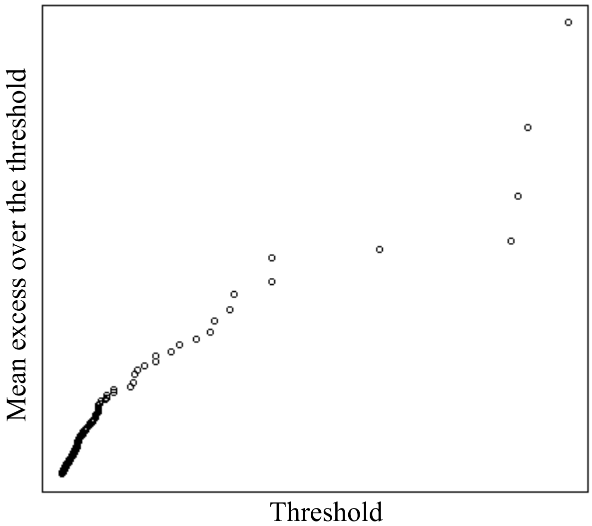


Figure 5: Mean-excess plot

3.5 Parameter estimation technique

We estimate the parameters of the distribution using both the maximum likelihood (ML) method and the probability weighted moments (PWM) method.[18] We have shown the results from both estimation methods, because estimated parameters differ considerably depending on the methods. At the same time, the parameters we use are mainly those obtained from the ML method,[19] considering that the number of data points is adequate when the threshold is set to 10 million Yen.[20]

3.6 Estimation results

Table 1 provides the estimation results. The results of two different estimation methods (ML, PWM) for four different thresholds (1 million, 10 million, 100 million, 1

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[18] The PWM method used in this paper employs the following procedure.
(i) Define the probability weighted moments of the GPD as:

$$M(p,r,s) = \mathrm{E}\left[x^{p} \{G(x)\}^{r} \{1 - G(x)\}^{s} \right],\ G(x)\text{: distribution function of the GPD}$$

(ii) Using the known relation of $M(1,0,s)=\beta/\{(s+1)(s+1-\xi)\}$ in (i) and inputting s=0 and s=1, a system of two equations is obtained. Estimates for β, ξ are solutions of the system. This assumes that ξ<1 (existence of average) and the estimation never exceeds 1.

[19] ML estimation is less robust when the sample size is small. It is often optimal when the sample size is deemed large, because of its good asymptotic properties — consistency, asymptotic normality and asymptotic efficiency — provided that the distribution assumption is correct. No ML method estimators for GPD parameters exist in a closed form, and numerical optimization is required.

[20] Hosking and Wallis (1987) states "unless the sample size is 500 or more, estimators derived by the method of moments or the method of probability-weighted moments are more reliable (than maximum likelihood estimation of the generalized Pareto distribution.)"

billion Yen) are shown. (Also see Figure 4 for ξ estimates for different thresholds.)

The ξ estimates using the ML method are very high, close to 1 or greater than 1 for all the thresholds, indicating a very heavy tail. Note that the GPD does not have a mean value (it is infinite) when ξ is greater than or equal to 1, and the ORC based on this estimate is expected to be very large. In particular, when the threshold is 100 million Yen (≈ € 633K) the ξ is especially high (1.543). The ORC calculated using this parameter estimation is so large that it may be considered to be what Nešlehová et al. (2006) calls a "ridiculously high capital charge."[21] However, this ξ (1.543) estimated with the threshold of 100 million Yen needs to be interpreted with caveats and its appropriateness should be tested against larger datasets given the small number of data points (88 data points) on which it is estimated, the large confidence interval (see Figure 4, top), and the fact that its estimation using the PWM method is 0.877.

On the other hand, ξ estimated with the threshold of 10 million Yen (0.973) appears to be within the reasonable range, considering that the ORC calculated using this ξ stays within the reasonable range. (See Section 4.3.2 "Reality of the capital estimates".)

Table 1: GPD parameter estimates

Threshold (Yen)	1 million (≈ € 6K)	10 million (≈ € 63K)	100 million (≈ € 634K)	1 billion (≈ € 6 million)
Number of losses ≥ threshold	7,151	883	88	13
ML method				
β	1.315	11.450	64.772	1,195.38
ξ	0.994	0.973	1.543	1.006
PWM method				
β	1.405	12.142	123.135	1,195.38
ξ	0.924	0.903	0.877	0.781

β: scale parameter, unit = 1 million Yen
ξ: shape parameter

[21] A GPD with a ξ greater than or equal to 1 does not have a mean (it becomes infinite) and the ORC based on that distribution is extremely large. Nešlehová et al. (2006) states that, "The transition from infinite second and finite first moment, say, to infinite first moment (the mean) is however a serious one and should be handled very carefully," and calls for consideration of possible reasons for such results, i.e., incorrect statistical inference or EVT assumptions, are not satisfied. Dutta and Perry (2006) have a negative view of applying GPD or extreme value theory to ORC calculations, partly based on the fact that GPD leads to unrealistically large amounts of ORC in many banks.

3.7 Goodness-of-fit of the common loss severity distribution

In this section, we examine the goodness-of-fit of the common loss severity distribution[22] estimated in Section 3.6 with the threshold of 10 million Yen using the ML method.

Table 2 shows the results of the goodness-of-fit tests of the common loss severity distribution to each of the individual banks.

- The null hypothesis is that the observed losses (the excesses over the 10 million Yen threshold) from each of the 18 banks originate from the common loss severity distribution. The alternative is that the observed losses (the excesses over the 10 million Yen threshold) from each of the 18 banks do not originate from the common loss severity distribution.
- Only 12 banks are tested, because six banks have fewer than five losses that are higher than the threshold (10 million Yen).
- Table 2 (top) reports the number of banks for which the null-hypothesis is rejected and the number of those for which it is not rejected. Table 2 (bottom) reports the P-values generated by the different tests for each bank.
- The rightmost column of Table 2 (bottom) shows how the common severity distribution predicts the empirically largest loss for each bank. A figure that is greater than 100% means that the common severity distribution underestimates the empirically largest loss, whereas a figure that is smaller than 100% means that the common severity distribution overestimates that loss.[23]

The tests that put more weight on the fit to the body — the Kolmogorov-Smirnov (KS) test and the Cramér-von Mises (CvM) test — do not reject the null hypothesis for any of the 12 banks tested. On the other hand, upper-tail Anderson-Darling tests (AD_{up} and AD^2_{up}),[24] which put more weight on the tail, give a lower P-value and reject the null hypothesis for nearly half of the banks tested.

The common severity distribution overestimates the empirically largest loss for each

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22 GPD with ξ= 0.973, β= 11.45 million Yen

23 This calculation is to show the performance of the common loss severity distribution by showing the difference between the prediction and the observed value for the largest figure that is available. Note that the percentile examined is much lower than the ORC (about 90% at the greatest), as the data collection period for each bank is between 3 and about 10 years.
The figure stands for the percentage of the following (a)/(b).
(a) The severity of the largest loss for each bank
(b) The prediction, which is provided by applying the percentile of (a) in empirical loss severity distribution for the bank* to the common loss severity distribution.
* The percentile of (a): $(100 - 100/2n)$, where n is the number of losses greater or equal to 10 million Yen, e.g., the percentile of the greatest loss is 90% when n is 5. The prediction is made based on the 90 percentile loss of the common loss severity distribution.

24 Please refer to Chernobai et al. (2005) for the details of the upper-tail Anderson-Darling tests.

of six of the banks and underestimates it for each of the other six, with a very large error in the case of underestimation.

Despite these observations, we have decided to use the estimates in Section 3.6 and assume that they are applicable to the individual banks' loss severity distributions in deriving the ORC formula in Section 4. This is based on the following considerations.

- The distribution estimate is the best one based on the available data and we expect that the parameters will not change much even when more data are accumulated. (Of course, the estimates need to be improved as more data are accumulated in the future.)
- The common severity distribution can be taken as the "median" of distributions for the 18 sample banks.
  - ✓ The common severity distribution overestimates the largest loss for each of six of the 12 tested banks and underestimates the largest loss for each of the other six. In other words, as far as it is judged from the empirically largest losses, the common loss severity distribution is located at the midway point among the 12 distributions of the individual banks.
- A definitive judgment is difficult to make about the tail part, because the available data is so limited.
  - ✓ The great difference between the predictions and the actual figures in the case of underestimation in Table 2 is largely attributable to the small number of data points. Therefore, it is expected that a longer observation period would reduce this difference. The fact that the margin of error shrinks significantly when a similar prediction is made on the consolidated data of all the 18 banks (the ratio shrinks to about 300% in this case) supports this expectation.

Table 2: Goodness-of-fit of the common loss severity distribution* to the datasets for individual banks**

Significance level		KS	CvM	AD_{up}	AD^2_{up}
5% Level	Not rejected	12	12	9	6
	Rejected	0	0	3	6
10% Level	Not rejected	12	12	8	5
	Rejected	0	0	4	7

	KS	CvM	ADup	AD^2up	Prediction error for the largest loss***
Bank 1	0.51	0.50	0.45	0.40	60%
Bank 2	0.89	0.84	0.52	0.37	45%
Bank 3	0.66	0.64	0.23	0.24	109%
Bank 4	0.68	0.58	0.38	0.22	32%
Bank 5	0.45	0.48	0.57	0.18	36%
Bank 6	0.32	0.35	0.41	0.08	70%
Bank 7	0.68	0.68	0.09	0.03	135%
Bank 8	0.29	0.29	0.67	0.02	24%
Bank 9	0.55	0.55	0.02	0.01	1,435%
Bank 10	0.30	0.31	0.00	0.00	1,283%
Bank 11	0.47	0.48	0.00	0.00	334%
Bank 12	0.44	0.44	0.23	0.00	109%

* GPD estimated in Section 3 (threshold: 10 million Yen, ML method)
** Losses greater than or equal to 10 million Yen (about 63K Euros)
*** (Severity of the largest loss predicted by the common severity distribution) / (severity of the largest loss that is observed)

Table 3 shows the results of the goodness-of-fit tests of the common loss severity distribution to the whole dataset of all the 18 sample banks lumped together. This test is effective in examining the appropriateness of choosing the GPD as the common loss severity distribution.

The null hypothesis is that the excesses over the 10 million Yen threshold follow the common loss severity distribution. The alternative is that the excesses over the 10 million Yen threshold do not follow the common loss severity distribution.

The KS test and the CvM test do not reject the null hypothesis. On the other hand, the AD_{up} and AD^2_{up} tests indicate lower P-values. Both of them do not reject the null hypothesis at a significance level of 5%, but both of them reject it at a significance level of 10%.

Table 3: Goodness-of-fit of the common loss severity distribution* to the dataset of all the banks**

	P-value	Significance level	
		5%	10%
KS	0.82	Not-Rejected	Not-Rejected
CvM	0.75	Not-Rejected	Not-Rejected
AD_{up}	0.07	Not-Rejected	Rejected
AD^2_{up}	0.08	Not-Rejected	Rejected

* GPD estimated in Section 3 (threshold: 10 million Yen, ML method)
** Losses greater than or equal to 10 million Yen (about 63K Euros)

The Quantile-quantile (QQ) plots in Figure 6 indicate similar results. The QQ-plot for the common loss severity distribution (left) shows a good fit for the body, but a poorer fit in the tail part. For comparison, a QQ-plot for a log-normal distribution[25] is indicated on the right. It shows a poorer fit in comparison to the GPD, especially in the tail region.

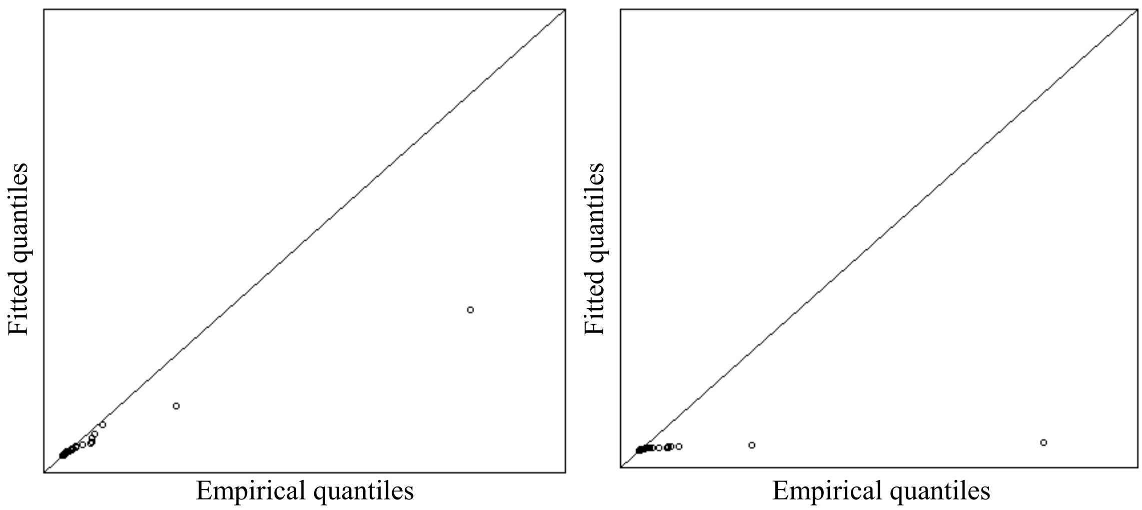


Figure 6: QQ-plots: GPD (left), log-normal distribution (right)

[25] Normal distribution is fitted to the logarithm of the losses that are greater than 10 million Yen (units: 10K Yen). The estimated parameters are: mean = 7.948, variance = 1.072.

3.8 Stability of the common loss severity distribution

3.8.1 Effects of a small number of huge losses

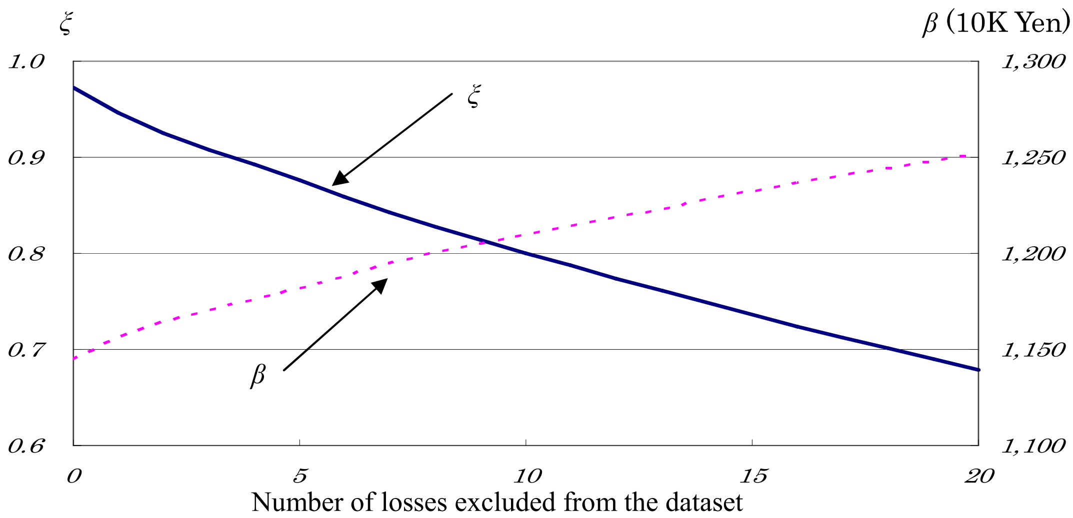
This paper estimates the shape of the common loss severity distribution based on a limited number of losses. This leads to concern that the shape of the loss severity distribution may be highly influenced by a small number of tail losses. To examine this, we follow the changes of the parameters of the GPD (ξ: shape parameter, β: scale parameter) as we discard or add big losses.

Note that changes of ξ are more important than those of β for us. The same change in the dataset results in greater change in ξ. In addition, changes in ξ have greater effects on the ORC than changes in β. When ORC is calculated using the formula proposed in this paper, ORC changes greatly in response to change in ξ, whereas it does not change so much in response to change in β.[26]

First, we observe the changes in the estimated ξ and β as we discard progressively smaller losses starting from the biggest loss, one by one, from the dataset consisting of the 18 banks. The results (Figure 7, top) show how the ξ decreases gradually, i.e., becomes less conservative, as the 20 biggest losses are discarded. When the biggest 20 losses are discarded, the ξ becomes as small as 0.679. On the other hand, the β increases gradually, i.e., becomes more conservative, as the 20 biggest losses are discarded, reaching 12.518 million Yen (about a 9% increase from the start) when the biggest 20 losses are discarded.

Figure 7 (bottom) shows the parameter changes when a single big loss is added. The ξ increases as single huge losses are added one at a time in increasing severity, from 10 billion Yen (about 63 million Euros) to 1 trillion Yen (about 6 billion Euros) in increments of 10 billion Yen. (Note that single losses are added to the original dataset, whereas in the former experiment up to 20 losses are excluded from the original.) The marginal increase in the ξ becomes smaller as the severity of the additional losses increases. For example, when a loss of 1 trillion Yen (about 6 billion Euros) is added, the ξ estimate is 1.012, just a little over 1. On the other hand, the β decreases gradually but only slightly, reaching only 11.274 million Yen when a loss even as large as 1 trillion Yen (about 6 billion Euros) is added.

[26] Suppose the ORC of a bank that incurred 10 losses greater than or equal to 10 million Yen (63K Euros) is calculated using the simple formula proposed in this paper. A 1% change in ξ results in a nearly 10% change in the ORC, while a 1% change in β results in only about a 1% change in ORC.



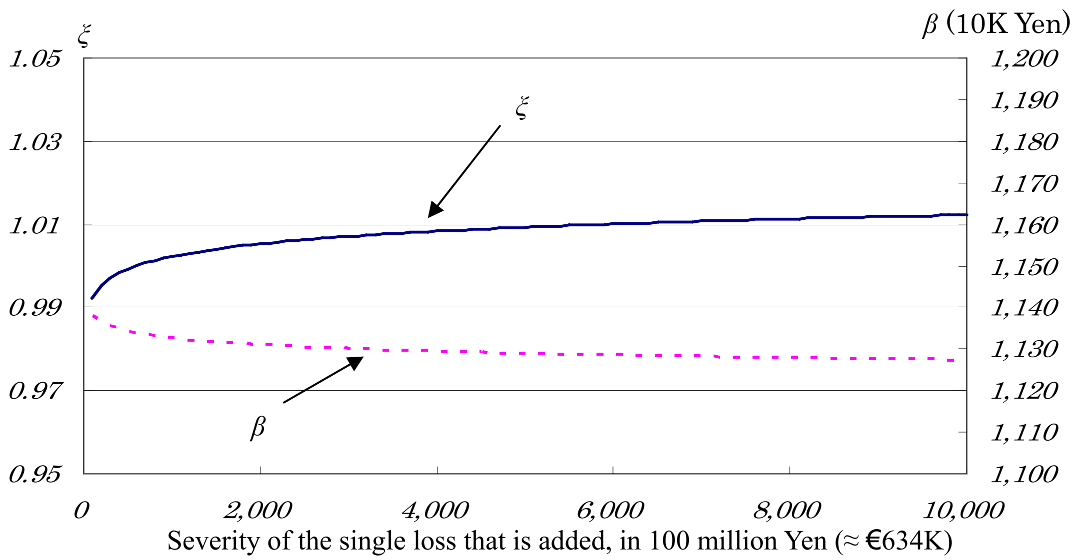


Figure 7: Parameter sensitivity to large losses:
starting from the largest loss, progressively smaller losses are excluded (top);
one large loss of a progressively larger size is added (bottom)

3.8.2 Effects of the dataset of banks with a larger number of losses

We have fitted a severity distribution to the pooled data from different banks without making any adjustments to the raw data. As a result, banks with more data tend to have a stronger influence on the shape of the curve. Some banks have more data because of their greater size (in general, the number of the losses increases roughly in proportion to the size of a bank) or because of their longer data collection period (data collection periods are different among banks in the sample dataset).

To provide some insight regarding this issue, we estimate the GPD parameters at the threshold of 10 million Yen (ML method) for the dataset of the nine medium sized-banks lumped together.[27]

The estimated parameters are ξ = 0.690 and β = 15.89 million Yen. They suggest a considerably safer risk profile than is indicated by the parameters for the whole dataset (ξ = 0.973, β = 11.45 million Yen). That is, if the parameters for the whole dataset were applied to the nine regional banks, they would lead to an overestimation of the ORC.[28]

Nevertheless, we assume that the parameters for the whole dataset are applicable to these medium sized banks and use them in deriving the ORC formula. This is because (i) the number of losses of 10 million Yen or greater from the nine banks is small and the estimates based on them are not considered to be very stable, (ii) the following sensitivity analysis shows that the parameters of the whole dataset are not completely unrealistic.

The outline of this sensitivity analysis is as follows.

- We have examined how the parameters change when a single big loss is added to the dataset of the nine regional banks lumped together;
- We have also examined how the ORC changes as the above changing parameters are used in the simple ORC formula. When the severity of the additional loss reaches about 2 billion Yen (≈ 13 million Euros), the parameters become ξ=0.947, β=15.10 million Yen. At that point, the ORC calculated using these parameters for the nine banks exceeds the ORC calculated using the parameters for the whole dataset as it is (ξ=0.973, β=11.45 million Yen);
- Adding a 2 billon Yen loss to the dataset is equivalent to a hypothetical single bank with the nine banks' average total assets (6.6 trillion Yen, about 41 billion Euros) incurring that loss every 36 years, i.e., 4 (average loss collection period) × 9.[29]

3.8.3 Stability of the distribution over time

The stability over time of the shape of the common loss severity distribution is impossible to assess without a long observation period. However, to provide some insight, we have given estimates of ξ and β using yearly datasets between the years 2002 and

[27] Those banks are called regional banks, of which none has total assets over 20 trillion Yen (127 billion Euros). The loss data from these banks account for only a small fraction of all the data. There are only 41 losses greater than or equal to 10 million Yen from these banks, which is about 4.6% of all the data points used for estimation (883 in number).

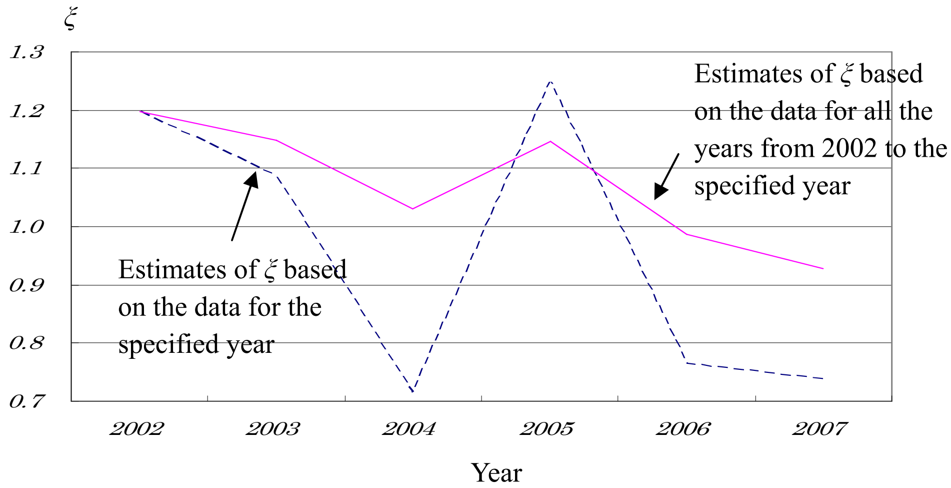
[28] The ORC for the following two sets of parameters are calculated using the simple formula proposed in this paper. A bank with 10 losses greater than or equal to 10 million Yen (63K Euros) is supposed.

- ORC using the parameters for the whole dataset: 91.8 billion Yen (582 million Euros).
- ORC using the parameters for the nine regional banks: 13.2 billion Yen (84 million Euros).

[29] While no public data are available for assessing the reality of the above point, we do not consider it excessively conservative, judging from our daily supervisory discussions with banks.

2007 in Figure 8 below. Estimates of parameters (ξ and β) based on yearly losses (broken line) and estimates of parameters based on all the losses accumulated up to the given year from 2002 (solid line) are indicated in the two graphs (top for ξ and bottom for β).

The parameters are considerably unstable on a one-year basis, which we think is mainly due to the small number of data points, rather than changes in the underlying risk profile.[30]



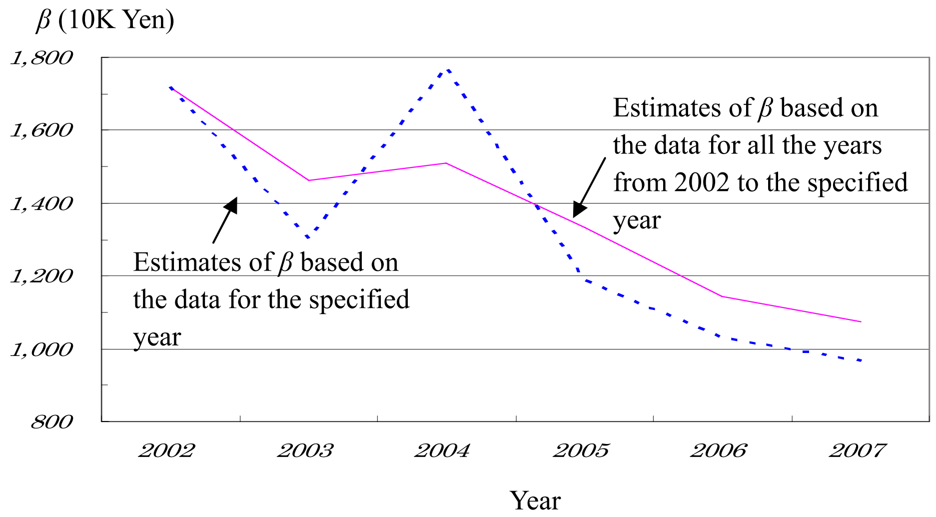


Figure 8: Parameter estimates by year: ξ (top), β (bottom)

[30] The parameter estimation for GPD is unstable when the data points are small in number. See Figure 4 for the relationship between ξ and the number of data points. In addition, according to our experience as regulators, we do not think risk profiles of banks change so much as the yearly changes in the GPD parameters indicate.

4. Simple ORC formula based on the common loss severity distribution

4.1 Simple ORC formula

We derive a simple ORC formula that calculates the individual 18 sample banks' 99.9% value-at-risk. We follow the loss distribution approach (LDA),[31] assuming that the common loss severity distribution estimated in Section 3 is applicable to the individual 18 banks.

We calculate the ORC analytically by applying the GPD estimated in Section 3 to the approximation that Böcker and Klüppelberg (2005) proposed (known as single loss approximation), thus eliminating the Monte Carlo simulation,[32] which is often used in applying the LDA.

Our formula is shown in the following box.[33] It has only a single variable: annual frequency of losses of a given threshold or greater (e.g., 10 million Yen).[34] The parameters of the assumed GPD are estimated in Section 3.

ORC = $(R - u + \beta/\xi)\cdot[1/(1-c)\cdot N_R]^{\xi} - (-u + \beta/\xi)$

where,

1. Variables for individual estimation
   - c = confidence level (e.g., 0.999 for a confidence level of 99.9%)
   - R = severity of losses for observation (e.g., 10 million Yen, about 63K Euros). R should be greater than or equal to u (the threshold for the GPD estimation, mainly 10 million Yen in this paper).
   - N_R = annual number of losses $\geq R$

2. GPD Parameters of the common loss severity distribution, estimated in Section 3.
   - ξ = shape parameter
   - β= scale parameter
   - u= threshold used to estimate the GPD parameters

When the specific parameters estimated in Section 3 (ξ: 0.973; β: 11.450 million Yen or about 73K Euros) are applied, the formula is:

ORC at a 99.9% confidence level (in 1 million Yen) = $(R{+}1.77) \cdot (1000\cdot N_R)^{0.973} - 1.77$

---

31 An annual aggregate loss distribution is generated by combining estimated loss frequency distribution and loss severity distribution.

32 Böcker and Klüppelberg (2005) showed that the ORC can be well approximated by the severity of a large single loss when the loss data distribution is heavy-tailed.

33 See Appendix 3 for the derivation of the formula.

34 Once the annual frequency of any given loss on the GPD is determined, one can know the frequency of any other losses on the GPD, including the loss that corresponds to the ORC. However, the shape of the loss severity distribution is determined only above the threshold for the estimation (i.e., u in the formula). Thus, N_R (loss frequency applied) needs to be the frequency of the losses greater than or equal to a given severity of R ($\geq u$).

When 100K Euros is used for R (the observation threshold for loss frequency), the formula can be written in euro terms as follows:

ORC at a 99.9% confidence level (in Euros) = $(100{,}000 + 11{,}201)\cdot(1000\cdot N_R)^{0.973} - 11{,}201$

where,

N_R stands for the annual frequency of losses $\geq R$ (=100K Euros).

4.2 Approximation error due to the single-loss approximation

Since our formula utilizes the approximation of Böcker and Klüppelberg (2005) (single-loss approximation), its approximation error should be examined.

We have done this by comparing ORCs calculated by our formula with the ones calculated by a conventional loss distribution approach (see Table 4 for the outline of the two calculation methods).[35]

The ORCs calculated by the two methods differ only between 0.6% and 1.6% for all the three cases, where the annual frequency of losses greater than or equal to 10 million Yen is 10, 100 or 1,000 (Table 5).

This means that our formula's error due to the single-loss approximation is negligible in practical use, even if the formula were applied widely to banks other than the 18 sample Japanese banks. No banks, not even the biggest overseas banks, are likely to incur more than 1,000 losses annually that are greater than or equal to 10 million Yen (63K Euros).[36]

Note that the above approximation errors assume specific conditions: the loss severity distribution is extremely heavy tailed (ξ is close to 1) and the confidence level is very high (99.9%). When these conditions are not satisfied, approximation errors can be bigger.[37]

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[35] Degen (2010) provides an analytical framework to estimate the accuracy of single-loss approximation. The estimated error according to their analytical framework is consistent with our simulation results.

[36] According to the medians for the 18 sample banks, the annual frequency of losses greater than or equal to 10 million Yen (about 63K Euros) per total assets of 100 billion Yen (about 634 million Euros) is about 0.021. This corresponds to an annual frequency of 21 for a bank with total assets of 100 trillion Yen (about 634 billion Euros).

LDCE2008 reports that the annual frequency of losses greater than or equal to 100K Euros per total assets of 1 billion Euros is about 0.19 (the median figure for the 118 banks, source: Table ILD9 in LDCE2008), which corresponds to an annual frequency of 190 for a bank with total assets of 1 trillion Euros.

[37] Please refer to Böcker and Sprittulla (2008) as well as Degen (2010) regarding the approximation error of single loss approximation.

Table 4: Comparison of our formula with conventional LDA

	(a) Our formula	(b) Conventional LDA for comparison
Outline	Analytical approximation of Operational Risk VaR ("Single loss approximation" by Böcker and Klüppelberg).	A frequency distribution and a loss severity distribution are combined in a Monte Carlo simulation to generate an aggregate loss distribution to calculate Operational Risk VaR.
Loss frequency distribution	No specific distribution is assumed.[38] Observed annual frequency (e.g., average for the past few years) is used directly.	A Poisson distribution is assumed
Loss severity distribution	"Common loss severity distribution," which is derived by fitting the GPD to the excess value over 10 million Yen. GPD parameters are estimated by the ML Method.	
Computation	Approximation by Böcker and Klüppelberg (2005). No simulation is used.	Simulation of 100,000,000 trials
Losses smaller than 10 million Yen	Theoretically, all the losses are included in the calculation	No loss smaller than 10 million Yen is included in the calculation.

Table 5: Single Loss Approximation errors

Annual frequency of losses	(a) ORC from our formula (unit: 100 million Yen)	(b) ORC from conventional LDA (unit: 100 million Yen)	(a)/(b)
10	918	924	99.4%
100	8,624	8,731	98.8%
1,000	81,039	82,380	98.4%

38 The approximation by Böcker and Klüppelberg assumes a specific loss severity distribution, but it does not assume a specific loss frequency distribution. This is because no tail information for frequency remains in the approximation formula and only the expected frequency remains. On the other hand, (b) in Table 4 assumes a Poisson distribution for frequency.

4.3 Performance of the formula

In this section, we assess the performance of the formula, utilizing the criteria that Dutta and Perry (2006) presented. The assessment is inevitably subjective, but we think that the overall performance is good enough to be used as a benchmark and a management tool.

The criteria Dutta and Perry (2006) presented are:

a. Good Fit — Statistically how well does the method fit the data?

b. Realistic — If a method fits well in a statistical sense, does it generate a loss distribution with a realistic capital estimate?

c. Well-Specified — Are the characteristics of the fitted data similar to the loss data and logically consistent?

d. Flexible — How well is the method able to reasonably accommodate a wide variety of empirical loss data?

e. Simple — Is the method easy to apply in practice?

4.3.1 Good Fit

In Section 3, we examined the goodness-of-fit of the common loss severity distribution. Although not perfect, it is as good as the models banks currently use. It can be implemented by individual banks at least as an approximation, as long as it is used with the caveat that goodness-of-fit will vary from bank to bank.

4.3.2 Reality of the capital estimates

To assess how realistic an ORC the formula gives, we have calculated the ORC by inputting the annual loss frequency per gross income of 100 billion Yen (634 million Euros) reported by the 18 sample banks,[39] which we compare with the BIA capital (15% of the gross income). The results (Table 6) show that the ORC calculated using this formula is about three quarters of the size of the BIA capital when the medians for the annual loss frequency of the 18 sample banks are applied. These medians can be construed as typical figures for the sample banks.

[39] The figures from each bank are listed in ascending order. The figure in the middle is the median (in our case, the average of the 9th and 10th figures, as there are 18 banks). The interquartile is a combination of the 25th percentile and 75th percentile. For example, the 25th percentile is the interior division of the ratio of 1 to 3 between the 5th and 6th figures (e.g., provided that the 5th figure is 40 and the 6th is 50, the 25th percentile is 42.5). In other words, we calculated the 25th and 75th percentile, assigning the 0th percentile to the minimum figure and the 100th percentile to the maximum figure.

Similar figures for global and regional (Australia, Europe, Japan, North America, Brazil/India) banks are available in the LDCE2008.

Whether the results are realistic or not is a matter of interpretation, but they are not far from the ORC figures calculated by banks themselves. Given that many ORC models, when they are being developed, produce apparently unrealistic figures, the formula works quite well in this respect.

Even when the same parameters are applied to banks outside of Japan,[40] the estimated ORCs remain realistic. The estimated ORCs can be about three times or four times as large as the BIA capital or equal to the gross income for some banks. This may look too conservative when compared to the ORCs currently calculated by banks but it may reflect the reality more accurately; indeed, anecdotal evidence reports losses of this size from time to time around the world. Some may argue that not many banks have failed because of operational risk, which also suggests that the formula is too conservative. However, a single operational risk loss corresponding to the 99.9% ORC from this formula is still not necessarily large enough to break a bank.

Table 6: ORC based on the frequency reported by the 18 sample banks (per gross income of 100 billion Yen, about 634 million Euros)[41]

Loss threshold	10 million Yen (63K Euros)	15.78 million Yen (100K Euros)	100 million Yen (634K Euros)
Annual frequency of losses ≥ loss threshold	1.147 (0.777–1.682)	0.8789 (0.4372–1.540)	0 (0–0.159)
ORC in billions of Yen (Corresponding BIA capital is 15 billion Yen)	11.1 (7.6–16.2)	12.8 (6.5–22.2)	0 (0–14.1)

Legend

0.1147	← median of the 18 banks
(0.777–1.682)	← the 25th percentile to the 75th percentile of the 18 banks

4.3.3 Well-Specified

This criterion deals with the question of whether the characteristics of the fitted data are similar to the loss data and logically consistent. In other words, we will be evaluating whether our formula simulates the real world reasonably well, and whether it behaves in

[40] As indicated in Section 5 "Applicability of the formula to other banks", the parameters are presumed to be about the same as the Japanese ones.

[41] The ORC calculated by our formula is not exactly proportional to the frequency of losses, as is shown in Section 4.3.3 "Well-specified." When the frequency of losses over the threshold becomes 10 times as large, the ORC becomes slightly less than 10 times as large, provided that ξ is a little smaller than 1. Because of this, the size of the bank (i.e., gross income) influences the results of the calculation: the ratio of the ORC to the gross income is smaller for a bank with the same annual frequency of losses per gross income, but with a larger gross income. (See also the next footnote.)

an intuitively and empirically correct manner.

In order to assess this, we have identified three characteristics of our formula's behavior. These characteristics are intuitively and logically consistent with our experience on operational loss data. We also find some aspects of them to be useful as a management tool, a fact which also suggests that the formula meets this criterion.

(i) ORC figures are almost proportional to the variable (annual frequency of losses)

The ORCs calculated using this formula are almost proportional to the loss frequency per year, when ξ is assumed to be 0.973 (Figure 9, Table 7). For example, when the annual frequency doubles, the ORC is 2.0 times as much. When the annual frequency triples, the ORC is 2.9 times as much.[42]

This appears to run counter to the notion that the tail losses determine the ORC. However, tail losses determine the ORC in this formula, too. What is unique about this formula is that it assumes a stable relationship between the annual loss frequency of small or medium sized losses and that of tail losses and that it estimates the severity of a single tail loss on this assumption.

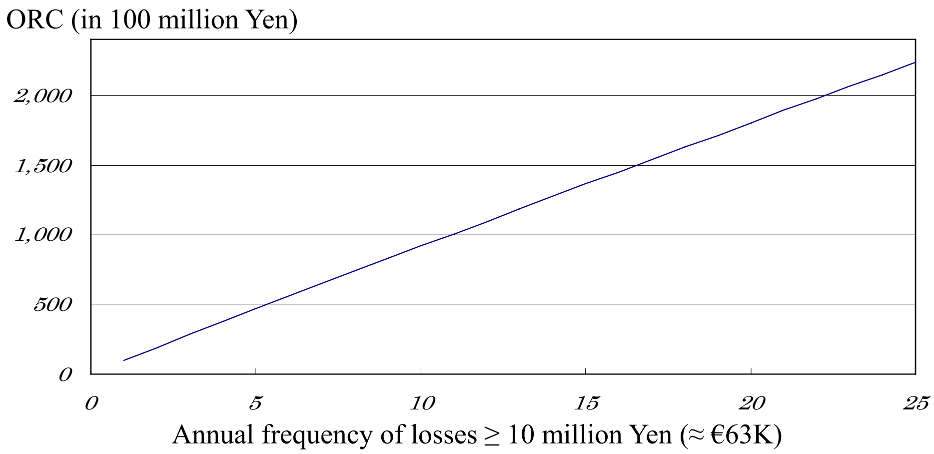


Figure 9: ORC changes with the change in the annual loss frequency

Table 7: Some figures in Figure 9

Number of Losses	5	10	15	20	25
ORC (in 100 million Yen)	467	918	1,362	1,801	2,238

42 More accurately, when the frequency is n times bigger, the ORC becomes n^{ξ}, provided that $-(-u + \beta/\xi)$ (or −1.77 specifically) is ignored.

(ii) ORC figures are insensitive to a small number of huge losses

On the flipside to (i), a single huge loss does not affect the ORC figures much; it only adds the number of losses (t) that are greater than or equal to the threshold. The increase in ORC is the same even if the loss is barely greater than the threshold.

(iii) ORC figures are very sensitive to ξ

Figure 10 indicates the changes in ORC in relation to changes in ξ. The ORC is set to 100 when ξ = 0.973. As the graph shows, the ORC calculated by this formula is very sensitive to ξ. For example, an increase of ξ from 0.973 to 0.993 increases a bank's ORC by about 18% when the bank annually incurs 10 losses of 10 million Yen or greater. On the other hand, a decrease of ξ from 0.973 to 0.953 decreases its ORC by about 15%. This very high sensitivity to ξ could pose challenges if an individual bank tried to determine its own specific ξ and account for its validity.

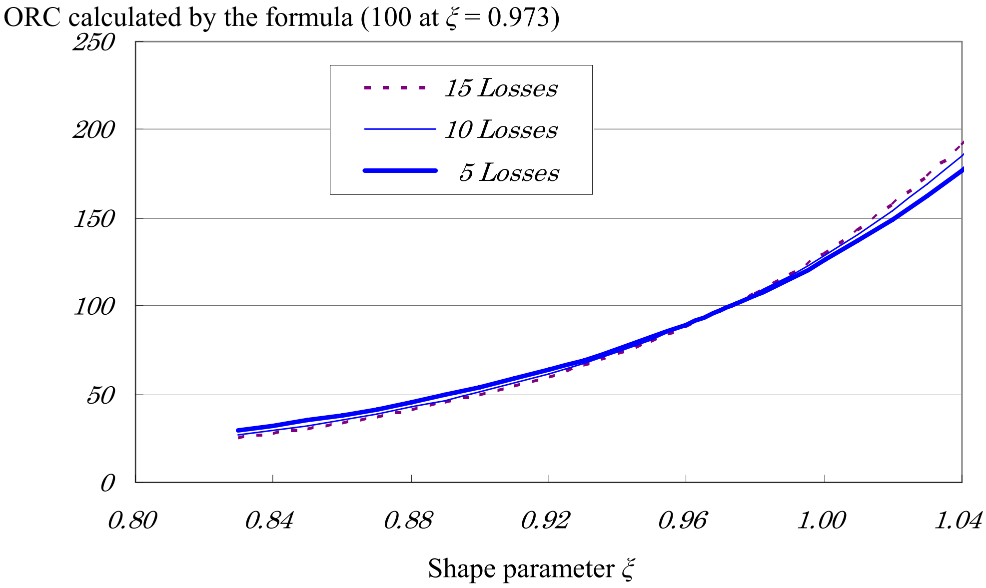


Figure 10: ORC changes with the change in ξ

Whether or not one believes that the formula is logical and intuitively correct depends to a large extent on whether one considers the characteristics of (i) and (ii) above to be realistic.

These characteristics, which are two sides of the same coin, are consistent with the view that the ORC is determined by tail losses, which can be estimated by the frequency of relatively small losses and that a bank's operational risk profile shows up through this frequency.

This view may appear exotic at first glance to theorists who are accustomed to thinking that tail losses determine the ORC, not the frequency of smaller losses. However, it is consistent with what we often hear from practitioners:

- Containing small losses can lead to reducing the probability of big losses (this idea has something in common with the broken window theory).
- A single huge loss does not necessarily indicate an abrupt increase in operational risk. On the other hand, change in the loss frequency is often the reflection of changes in underlying business environment and internal control factors (BEICFs) or loss data collection procedures.
- The occurrence of losses is relatively easy to control, but their severity is hard to control (in many cases, it depends on chance).

In addition, the characteristics of (i) and (ii) above are useful for many aspects of risk management, which supports our view that the formula is realistic and intuitively correct.

The first useful aspect is the stability of the ORC. This stems from the fact that the ORC changes in proportion to the loss frequency, which is usually much more stable than the severity of losses; a single huge loss that occurs by chance does not cause too much fluctuation in capital estimates.[43]

The second aspect is the near additivity of the risk amount, i.e., the risk of a bank is the same as the total amount of the risks for the different parts of the bank. This also is due to the near-linear way the ORC, as calculated by this formula, behaves in relation to the loss frequency. Thanks to this additivity, it is easy to calculate the ORC in the case of a merger or division among banks or businesses. In addition, the allocation of the ORC is easier. Banks can determine reasonable figures by allocating the ORC simply according to the number of losses above a certain threshold if they assume that the loss severity distributions are identical across business lines. See Appendix 2 for an analysis at the business-line level.

The third aspect is that these characteristics provide risk managers with a good reason to reduce the number of relatively small losses, in the hope that it will have the effect of reducing the eventuality of huge losses. While the goal of risk management is preventing huge losses, this sets an attainable goal in day-to-day risk management.

[43] It is assumed that a huge loss does not alter the shape of the common loss severity distribution. This assumption is reasonable, given the result of the sensitivity analysis conducted in Figure 7 (bottom).

4.3.4 Flexibility

The formula is applicable to any bank over time as long as the common loss severity distribution is reasonably fitted; the difference in risk profile among banks is assumed to be reflected in the loss frequency.

What matters is the goodness-of-fit of the common loss severity distribution to banks other than the 18 sample banks. This issue is dealt with in the next section (5. "Applicability of the formula to other banks").

4.3.5 Simplicity

This formula is very simple, which also makes the whole process transparent. It uses only one variable, the annual loss frequency, which is easy to handle. On top of that, the formula has the advantage of always giving the same ORC for the same variable because it does not use Monte Carlo simulation.

The challenge in implementation lies in the data collection: the same criteria in loss data collection as the LDCE2008 is required to apply the formula as it is.

5. Applicability of the formula to other banks

So far, we have derived an ORC formula based on the loss severity distribution fitted to the 18 sample Japanese banks' losses. However, it is highly likely that our formula is applicable to other Japanese banks as well and even to overseas banks without the need for many changes. We can say this because the results of various loss data collection exercises and empirical studies on ORC measurement suggest that the loss severity distribution we estimated on the 18 sample Japanese banks fits well to a wider range of banks.

We expect that this will be confirmed by the accumulation of loss data in the future, but in the meantime, this section shows the available published data and the previous research.

5.1 Results of various loss data collection exercises

Some results of loss data collection exercises are available to obtain a rough shape of loss severity distribution. The shapes of the loss severity distribution inferred from the published figures of the LDCE2008 or U.S. data collection exercise[44] (2004 Loss Data

[44] Federal Reserve System, Office of the Comptroller of the Currency, Office of Thrift Supervision, Federal Deposit Insurance Corporation (2005)

Collection Exercise for Operational Risk; "US LDCE2004" hereafter) are shown in Figures 11, 12 and 13. All of them suggest a GPD with a ξ of about 1.

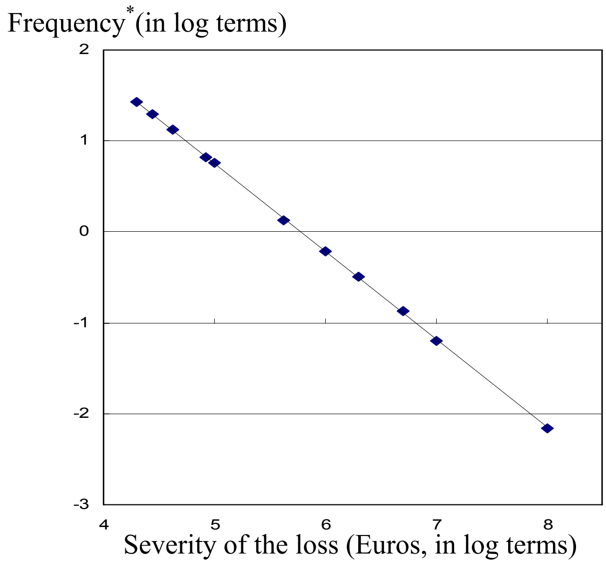
Figure 11 plots the frequency of losses per gross income of 1 billion Euros for various severity thresholds in a similar manner as Figures 1 to 3. The means or medians across the 118 banks around the world that reported in the LDCE2008 are used.[45]

The plotted data points are approximately on a straight line. The slope of the line is approximately −0.97 when a simple regression is applied. Given that the slope is roughly $-1/\xi$ (ξ is the shape parameter of the GPD), ξ for the LDCE2008 banks is inferred to be around 1.03 when the loss is between 20K and 100 million Euros (the maximum amount of loss whose frequency is available), which is not far from the figure estimated for the 18 Japanese banks (0.973).

Figure 12 plots losses in the same manner by region using the figures available in LDCE2008 (source: Table ILD7 in LDCE2008). It shows great similarity in loss severity distributions, a nearly straight line with a slope of −1 on a double-log plot, among various regions (Australia, Europe, Japan and North America) except for Brazil/India. Indeed, losses from regions other than Brazil/India are difficult to distinguish in the plot.

Figure 13 also plots losses in the same manner using the figures for US banks available in US LDCE2004. The plot is also very similar to the previous ones.

[45] "Mean" and "Median" in Figure 11 are used in the following ways.
Mean: Frequency is calculated for the dataset for all the losses from all the 118 sample banks lumped together; 118 banks are treated as if they were merged into one single bank.
Median: Frequency is the median across the individual figures for each of the 118 sample banks.
The mean is used in the plot when it is available, otherwise the median is used.



* Annual loss frequency ≥ severity indicated on the *x*-axis
(per gross income of €1 billion, in log terms)

Figure 11: Severity distribution for 118 banks around the world

Table 8: Actual value of the data points for Figure 11

(before their logarithms are taken)

Severity (Euros) (*x*-axis)	Frequency (*y*-axis)	Source*
20,000	26.60	ILD9
27,632	19.95	ILD7
41,916	13.30	ILD7
82,608	6.65	ILD7
100,000	5.84	ILD6
418,400	1.33	ILD7
1,000,000	0.61	ILD6
2,000,000	0.32	ILD6
5,000,000	0.13	ILD6
10,000,000	0.06	ILD6
100,000,000	0.007	ILD6

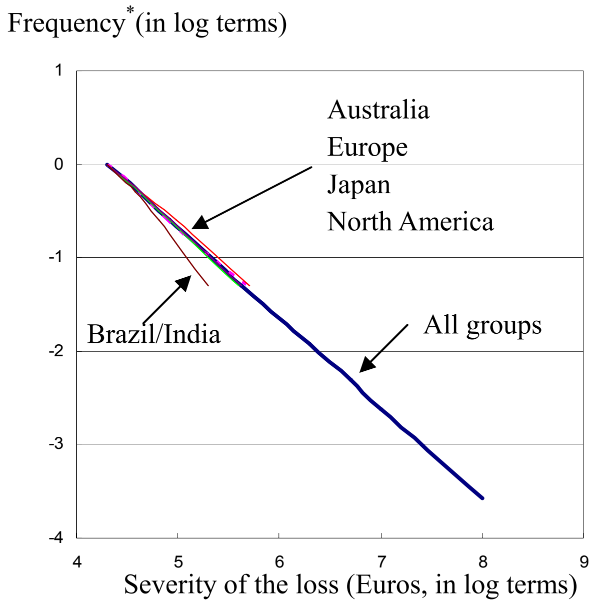
* Table name in the LDCE2008

The figures in the table are obtained from the published figures in the LDCE2008 as follows.

Figures from ILD9: The loss frequency per gross income of €1 billon (cross-bank medians) reported in ILD9.

Figures from ILD6: Calculated from the aggregated loss frequency figures for all the banks reported in ILD6.
(i) Divide (# of losses ≥ specific severity) by (# of losses ≥ €20K).
Example: In the case of losses that are €100 million or greater, divide 41 (losses ≥ €100 million) by 155,713 (losses ≥ €20K).
(ii) Multiply (i) by the annual frequency of losses ≥ €20K per gross income of €1 billion. The result is the frequency of the losses specified in (i).
Example: In the case of losses that are €100 million or greater: 41/155,713 × 26.6 = 0.007

Figures from ILD7:
(i) Obtain the severity of the loss for a specific percentile point (e.g. €418,400 for fifth percentile) from ILD7.
(ii) Multiply (i) by the annual frequency of losses ≥ €20K per gross income of €1 billion. The result is the frequency of the losses specified in (i) (e.g., 26.6 × 5% = 1.33)



$$* \text{ Frequency} = \frac{\text{number of losses}(\geq \text{severity on the } x\text{-axis})}{\text{total number of losses}(\geq 20\text{K Euros})}$$

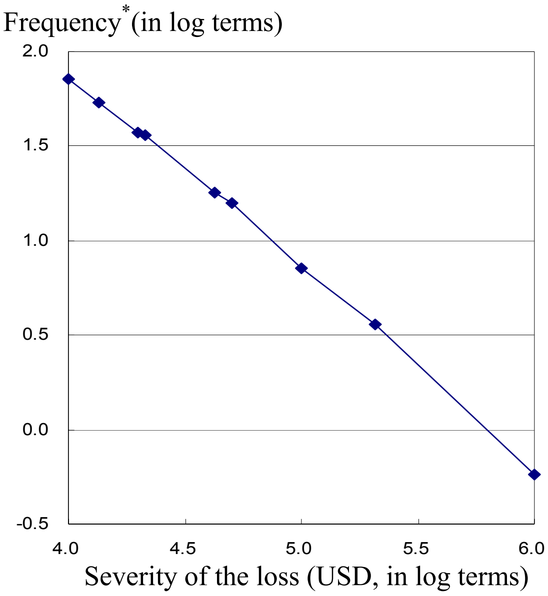
Figure 12: Severity distribution by regions

Table 9: Actual value of the data points for Figure 12

(before their logarithms are taken)

Percentile points*	**100%**	**75%**	**50%**	**25%**	**5%**
Frequency	1.00	0.75	0.50	0.25	0.05
Australia	20,000	28,883	41,781	75,966	476,708
Europe	20,000	27,341	41,765	82,004	400,000
Japan	20,000	27,890	42,357	95,093	511,815
North America	20,000	27,900	43,234	84,462	425,314
Brazil/India	20,000	26,472	37,962	65,100	201,416

* Percentile points among the losses over €20K



* Annual number of losses ≥ severity on the *x*-axis
(per gross income of $1 billion, in log terms)

Figure 13: Severity distribution observed in US 2004 LDCE

Table 10: Actual value of the data points for Figure 13
(before their logarithms are taken)

Severity ($) (*x*-axis)	Frequency (*y*-axis)	Source*
10,000	72.035	Table Appendix 3a, Table 8
13,436	54.026	Table 7a
20,000	37.530	Table 8
21,277	36.017	Table 7a
42,155	18.009	Table Appendix 3a
50,000	15.704	Table Appendix 3a
100,000	7.170	Table 8
206,492	3.602	Table 7a
1,000,000	0.576	Table 8

* Table name in the US LDCE2004

5.2 Empirical studies

Many empirical studies on ORC have estimated shape parameters around 1 for GPD (Table 11).

Table 11: ξ estimates of empirical studies on ORC with loss data

(Typical figures from each study are indicated).

Data	Unit of measure	ξ estimates	Note
6 US banks, each for one year (LDCE2002)	Enterprise level for each bank	Estimates for six banks Minimum: 0.87, maximum: 1.28 average: 1.01, median: 0.98	de Fontnouvelle et al. (2004)
89 banks around the world, each for one year (LDCE2002)	All the losses for all the sample banks broken down into 8 business lines	Estimates for eight business lines. Minimum: 0.85, maximum: 1.39 average: 1.13, median: 1.18	Moscadelli (2004)
7 US banks	Enterprise level for each bank	Estimates for seven banks when the threshold is set so that 10% of the data are in the tail Minimum: 0.89, maximum: 1.20 average: 1.02, median: 1.01	Dutta and Perry (2006)
Japanese banks (sample size not specified), for 10 years	All the losses for all the sample banks	Estimates when the threshold is set to the minimum loss amount in the dataset Maximum likelihood method: 1.10 PWM method: 0.98	Mori et al (2007)
An Australian bank for 9 years	Enterprise level	Maximum likelihood method: 1.04 PWM method: 0.81	John Evans SF Fin et al. (2008)

6. Final words

We have examined the hypothesis that the common loss severity distribution for operational risk loss data fits reasonably well to a wide range of banks. Assuming this hypothesis, we have derived a simple formula that can be used as a benchmark for ORC.

In spite of the formula's simplicity, it is also risk sensitive and has other good qualities as well, i.e., it is well specified and it produces reasonably realistic results. The fact that the formula seems to work well in practical applications suggests that the common loss severity distribution hypothesis is effective in approximating banks' loss severity distributions.

Of course, the proposed formula comes with several challenges and cannot be used as an AMA model as it is. First, it is partly true that recent BEICFs are not reflected. The formula stands on the assumption that BEICFs are reflected in the loss frequency and that the loss severity distribution retains its shape over time and across banks. However, the latter assumption is for the purpose of the simplicity of the approximation and to circumvent the possibility that data for individual banks are scarce. Some banks will not wish to maintain it in their more sophisticated models.

In addition, the formula may give banks distorted incentives. The ORC figures derived from this formula are insensitive to a small number of huge losses, which may encourage banks only to reduce the number of relatively small losses rather than contain huge losses directly.

For the above reasons, it is completely reasonable for individual banks to move toward more sophisticated models that reflect their risk profiles in a more granular way, by, for example, utilizing scenario analyses and BEICF measures.[46]

When it comes to implementation, measurement of the frequency of losses poses a number of challenges, the first of which is determining the severity of losses (R in the formula) in order to measure the annual loss frequency (N_R). If the loss severity distribution for an individual bank is exactly the same as the common loss severity distribution, any R (though R must be greater than or equal to the threshold used to estimate the GPD parameters) gives the same amount of ORC. However, since few banks' actual loss severity distributions can be expected to have exactly the same shape as the

[46] For example, the following are possible measures:

- Using scenario analysis to forecast possible events in the future. This adds some conservatism to the ORC and also gives banks incentives to take measures to contain possible events;
- Estimating a huge loss for events that can be more accurately estimated* through other measures and comparing this with the result given by the formula, and adding appropriate conservatism to the result.

  * Damage by earthquakes can be estimated more accurately by utilizing the knowledge from seismic science (see, for example, Kanemori (2006)).

common severity distribution, this formula would most likely give a different ORC for a different *R*. For this reason, banks would be held accountable for their choice of *R*.

Once *R* is determined, the next challenge is how to determine the specific frequency of losses (N_R) used in the formula, e.g., observation years or the calculation of N_R from the observations.[47] This is not an easy task and cannot be done automatically. N_R should be the best prediction for the coming year based on the latest BEICFs, but it should not be too subjective.

Banks with few or no losses greater than or equal to 10 million Yen (63K Euros) face another unique implementation challenge: they have no frequency figures for input into the formula. They may need to develop a reasonable model to forecast the number of losses over 10 million Yen and use that number as input for the formula.[48]

The parameters of the formula are determined only on the basis of the 18 sample Japanese banks. However, as we have seen in Section 5, based on a variety of public information we think that the estimated parameters will not change much when the sample data are extended to a wider group of banks, be they foreign or domestic.

Further analysis on additional data is necessary to confirm this. We hope that our formula is validated against a wider range of datasets. We also hope that this paper encourages quests for a better benchmark for ORCs, possibly standing on different and novel ideas.

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[47] The following may prove to be issues in determining the frequency to be used in the formula:

- Observation periods for the frequency, e.g., the most recent three years, five years or all the years when the frequency is known;
- The calculation of the observed frequencies over time, e.g., average, weighted average or the maximum;
- The effects of a new or a discontinued business;
- The extent to which the frequency to be used in the formula may be adjusted subjectively, e.g., determining the frequency by extrapolating the most recent trend;
- How much conservatism should be incorporated when the observation period is too short.

[48] Inputs for such a model may include:

- the frequency of losses smaller than 10 million Yen;
- the frequency of near misses;
- other BEICFs.

Using these measures also provides incentives to properly control these figures.

Appendix 1 Non-parametric tests for the commonality of loss severity distributions

In this appendix, we perform a non-parametric test to explore the commonality of loss severity distributions among 18 sample banks. We conduct the Kruskal-Wallis test[49] to test the null hypothesis that the loss severity distribution is identical across 18 sample banks versus the alternative that losses in at least one bank do not follow the same loss severity distribution (Figure A1-1).

When the significance level is set at 5 or 10%, the null hypothesis is not rejected in most of the cases if the threshold for the losses to be tested is set at 15 million Yen (about 95K Euros) or higher. This result gives one of the rationales for assuming that the common severity distribution can be used as an individual banks' loss severity distribution.

The test is conducted against losses greater than or equal to 100 different thresholds between 1 million Yen (about 6K Euros) and 100 million Yen (about 634K Euros), in 1 million Yen increments. For all the thresholds that are below 15 million Yen (about 95K Euros), the null hypothesis is rejected at the significance level of 5%. For most of the thresholds between 15 million Yen and 100 million Yen, the null hypothesis is not rejected at the significance level of 5 or 10%.

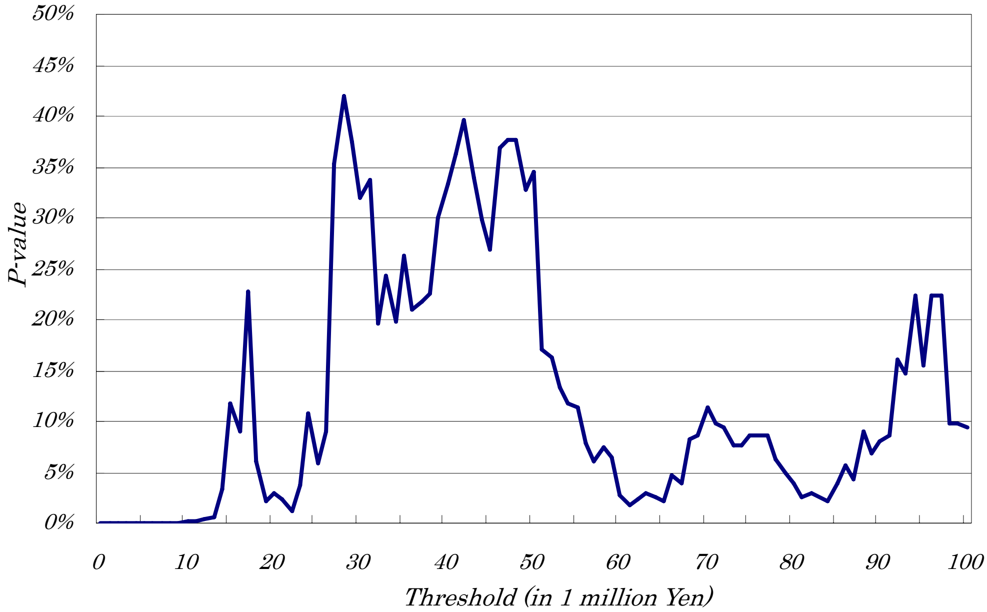

Figure A1-1: Kruskal-Wallis test

[49] De Fontnouvelle et al. (2005) conducted a similar test (Kruskal-Wallis test) on big U.S. banks and arrived at a similar result: "we cannot reject the hypothesis that the distribution of losses is the same across large firms."

Appendix 2 Analysis at the level of business line or event type

This appendix examines loss severity distributions for individual Basel business lines or event types.[50] Some business lines or event types have a very small number of data points and the results of this appendix should be interpreted with caution.

A2.1 Commonality of severity distributions for individual business lines or event types

We conduct Kruskal-Wallis tests to test the commonality of distributions across individual business lines or across event types in the following manner.

- Datasets tested: datasets in which losses from the 18 sample banks are first lumped together and then broken down by the eight individual business lines or the seven event types.
- Null hypothesis for business lines: loss severity distributions are identical across eight business lines when the 18 banks' losses are lumped together by business line. (The alternative: losses for at least one business line do not come from the identical distribution.)
- Null hypothesis for event types: loss severity distributions are identical across seven event types when the 18 banks' losses are lumped together by event type. (The alternative: losses for at least one event type do not come from the identical distribution.)
- Tests conducted: for both the null hypotheses, we test against losses greater than or equal to 100 different thresholds from 1 million Yen (about 6K Euros) to 100 million Yen (about 634K Euros), in increments of 1 million Yen.

Figure A2-1 reports the test results. The p-values for the across-business-line tests are greater than 10% for most of the thresholds of 8 million Yen or higher (about 51K Euros), so we cannot reject the null hypothesis that the loss severity distribution of losses is the same across business lines for losses of 8 million Yen or greater. On the other hand, the p-values for the across-event-type-tests are under 5% for most of the thresholds, requiring us to reject the null hypothesis for most of the thresholds.

[50] See Table A2-5 for the abbreviations of the Basel business lines and event types.

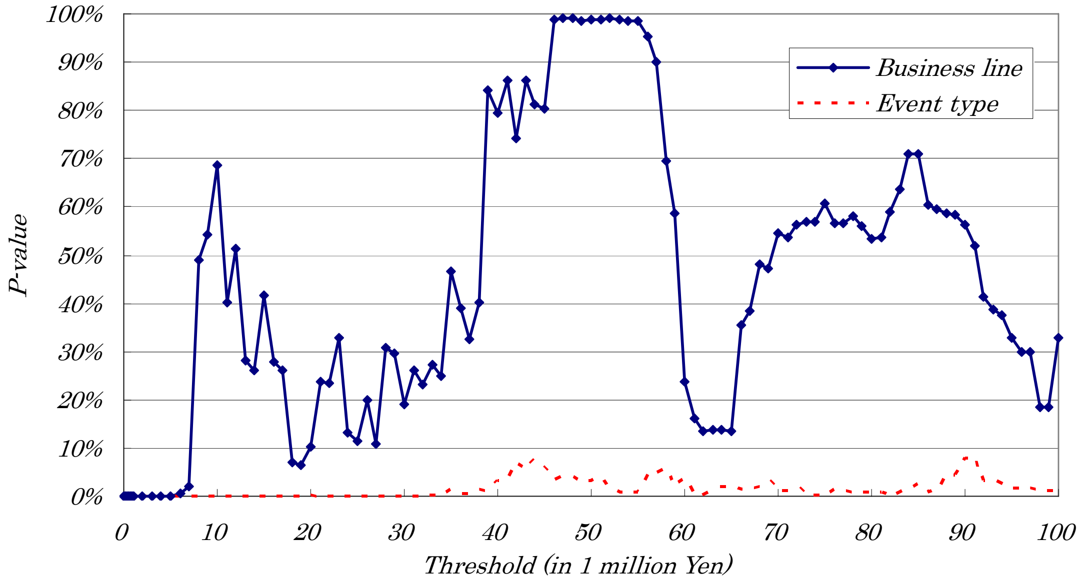

Figure A2-1: Kruskal-Wallis test across business lines or event types

A2.2 Loss severity distribution for individual business lines

Here we estimate a severity distribution for each of the eight datasets corresponding to eight individual business lines for all the 18 banks lumped together. Assuming that each distribution is the GPD, we estimate GPD parameters with a threshold of 10 million Yen (about 63K Euros) using the ML method. We then conduct goodness-of-fit tests for the estimated distributions.

The results are indicated in Table A2-1. ξs for individual business lines range from 0.56 to 1.38, indicating a very different shape of distribution by business line.

However, it is reasonable to suppose that, as more data accumulate, the ξs for individual business lines may converge. We expect this, given that the divergence in ξs might be the result of the small sample size for each individual business line. In addition, the results presented in Section A2.1 in this appendix and in Table A2-2 below suggest that the loss severity distributions across business lines are likely to be identical.

Note that the arithmetic average of ξs for individual business lines is 0.96 (the weighted average, by the number of losses is 1.01), which is near the ξ estimate for the whole dataset (0.973).

Table A2-1 Distribution estimates and their goodness-of-fit for business lines

	BL1	BL2	BL3	BL4	BL6	BL7	BL8
Number of losses	11	44	224	480	38	34	37
β (10K Yen)	1,485	1,105	1,340	1,032	1,195	1,628	1519
ξ	1.38	1.17	1.01	0.92	1.21	0.56	0.84
KS	0.73	0.85	0.76	0.86	0.88	0.91	0.88
CvM	0.83	0.85	0.68	0.81	0.93	0.88	0.83
AD_{up}	0.56	0.01	0.18	0.10	0.17	0.18	0.29
AD^2_{up}	0.56	0.01	0.35	0.08	0.15	0.31	0.31

* No figures for BL5 are shown, as the sample size is too small (less than 10). No figures for the losses are indicated where the business line is not known.
** P-values for goodness-of-fit tests (KS, CvM, AD_{up}, and AD^2_{up}) are reported.

Table A2-2 reports the results of the goodness-of-fit tests of the common loss severity distribution for the integrated dataset of the 18 sample banks (the GPD with a ξ of 0.973) to the business-line breakdown thereof. The null hypothesis is that the observed losses for individual business lines of the integrated dataset of the 18 banks follow the common loss severity distribution (the alternative: the observed losses for individual business lines of the integrated dataset of the 18 banks do not follow the common loss severity distribution). The tests that put more weight on the body fit (KS, CvM) do not reject the null hypothesis for any of the eight business lines, whereas the tests that put more weight on the tail fit (AD_{up} and AD^2_{up}) reject the null hypothesis for some business lines.

Table A2-2 Goodness-of-fit of the common loss severity distribution at the business-line level

	BL1	BL2	BL3	BL4	BL6	BL7	BL8
KS	0.43	0.88	0.63	0.52	0.91	0.85	0.75
CvM	0.61	0.96	0.64	0.53	0.88	0.79	0.71
AD_{up}	0.01	0.00	0.18	0.17	0.05	0.27	0.32
AD^2_{up}	0.00	0.00	0.11	0.09	0.02	0.29	0.17

* No figures for BL5 are shown, as the sample size is too small (less than 10). No figures for the losses are indicated where the business line is not known.
** P-values for goodness-of-fit tests (KS, CvM, AD_{up}, and AD^2_{up}) are reported.

A2.3 Loss severity distribution for individual event types

Here we estimate a severity distribution for each of the seven datasets corresponding to seven individual event types for all the 18 banks lumped together. Assuming that each distribution is the GPD, we estimate GPD parameters with a threshold of 10 million Yen (about 63K Euros) using the ML method. We then conduct goodness-of-fit tests for the estimated distributions.

The results are indicated in Table A2-3. ξs for individual event types range from 0.32 to 1.23, indicating a very different shape of distribution by event types.

However, similar to estimates made at the business line level, it is reasonable to suppose that the ξs for individual event types may converge as more data are accumulated. We expect this, given that the divergence in ξs might be the result of the small sample size and in view of the results presented in Table A2-4 below, which do not always reject the hypothesis that the observed losses for individual event types follow the common loss severity distribution.

Note that the arithmetic average of ξs for individual business lines is 0.92 (weighted average by the number of losses is 0.89).

Table A2-3 Distribution estimates and goodness-of-fit for event types

	ET1	ET2	ET3	ET4	ET5	ET6	ET7
Number of losses	61	106	23	153	23	36	479
β (10K Yen)	2,907	1,056	1,024	1,866	1,233	919	990
ξ	0.97	0.84	1.12	1.17	0.32	1.23	0.80
KS	0.84	0.66	0.85	0.82	0.75	0.56	0.99
CvM	0.79	0.57	0.83	0.86	0.70	0.58	0.99
AD_{up}	0.19	0.28	0.04	0.22	0.09	0.13	0.00
AD^2_{up}	0.49	0.08	0.09	0.31	0.24	0.09	0.00

* No figures for the losses are indicated where the event type is not known.
** P-values for goodness-of-fit tests (KS, CvM, AD_{up}, and AD^2_{up}) are reported.

Table A2-4 reports the results of the goodness-of-fit tests of the common loss severity distribution for the integrated dataset of the 18 sample banks (the GPD with a ξ of 0.973) to the event-type breakdown thereof. The null hypothesis is that the observed losses for individual event types of the integrated dataset of the 18 banks follow the common loss severity distribution. (The alternative: the observed losses for individual event types of the integrated dataset of the 18 banks do not follow the common loss severity distribution.) The tests that put more weight on the body fit (KS, CvM) do not reject the null hypothesis for any of the seven event types, whereas the tests that put more weight on the tail fit (AD_{up} and AD^2_{up}) reject the null hypothesis for some event types.

Table A2-4 Goodness-of-fit of the common loss severity distribution at event-type level

	ET1	ET2	ET3	ET4	ET5	ET6	ET7
KS	0.50	0.53	0.91	0.57	0.44	0.69	0.53
CvM	0.49	0.53	0.88	0.57	0.41	0.66	0.53
AD_{up}	0.05	0.28	0.00	0.08	0.42	0.01	0.08
AD^2_{up}	0.01	0.06	0.00	0.01	0.04	0.00	0.00

* No figures for the losses are indicated where the event type is not known.
** P-values for goodness-of-fit tests (KS, CvM, AD_{up}, and AD^2_{up}) are reported.

Table A2-5 Business lines and event types

These tables indicate the abbreviations of the Basel business lines and event types.

Business lines

	Basel Level 1 Business Lines
BL1	Corporate Finance
BL2	Trading & Sales
BL3	Retail Banking
BL4	Commercial Banking
BL5	Payment and Settlement
BL6	Agency Services
BL7	Asset Management
BL8	Retail Brokerage

Event types

	Basel Level 2 Event Types
ET1	Internal Fraud
ET2	External Fraud
ET3	Employment Practices and Workplace Safety
ET4	Clients , Products & Business Practices
ET5	Damage to Physical Assets
ET6	Business Disruption and System Failures
ET7	Execution , Delivery & Process Management

Appendix 3 Derivation of the simple ORC formula

Approximate ORC (one-year holding period, confidence level c) can be calculated with the following simple formula

$$\mathrm{ORC} = (R - u + \beta/\xi) \cdot [1/(1-c) \cdot N_R]^{\xi} - (-u + \beta/\xi)$$

where

c: Confidence level (e.g., 0.999 [=99.9%] according to regulations)
R: Threshold of loss amount (e.g., 20 million Yen), $R \geq u$
N_R: Annual number of losses $\geq R$
ξ: Shape parameter of the GPD
β: Scale parameter of the GPD
u: Threshold of the losses used in the estimation of the GPD (10 million Yen)

The formula is derived through the single loss approximation proposed by Böcker, C. and Klüppelberg as follows.

Let X be a random variable for loss severity and F be a loss severity distribution. The excess distribution over a large threshold u is given by

$$\begin{aligned} F_u(x) &= P(X - u \leq x | X > u) \\ &= \frac{F(x+u) - F(u)}{1 - F(u)} \end{aligned}.$$

The excess distribution F_u describes the distribution of $x = (X - u)$. (x: the excess loss severity over the threshold u when the loss severity X exceeds u.)

Let the distribution function of the GPD be given by

$$G_{\xi,\beta}(x) = 1 - \left(1 + \frac{\xi}{\beta} x\right)^{-\frac{1}{\xi}}$$

where ξ is a shape parameter and β is a scale parameter.

The theorem of Pickands-Balkema-de Haan states that the following equation holds true when u is large enough.

$$F_u(x) \cong G_{\xi,\beta}(x)$$

This means that we can approximate the excess distribution F_u by GPD $G_{\xi,\beta}$ if we set appropriate parameters ξ and β for a large threshold u even when we do not know the original (underlying) distribution F.

Let $\overline{F}(x) = 1 - F(x)$, $\overline{G}_{\xi,\beta}(x) = 1 - G_{\xi,\beta}(x)$ for F (loss severity distribution) and $G_{\xi,\beta}$ (GPD with parameters ξ and β), then,

$$\begin{aligned}\overline{F}(x) &= P(X > u)P(X > x | X > u) \\ &\cong \overline{F}(u)\overline{G}_{\xi,\beta}(x-u) \\ &= \overline{F}(u)\left(1 + \frac{\xi}{\beta}(x-u)\right)^{-\frac{1}{\xi}}\end{aligned}$$

Eliminating $\overline{F}(u)$ utilizing $\overline{F}(R) = \overline{F}(u)\left(1 + \frac{\xi}{\beta}(R-u)\right)^{-\frac{1}{\xi}}$ for given $R(\geq u)$, we obtain the following equation for any given p ($p = F(x)$, $0 \leq p \leq 1$).

$$\begin{aligned}x &= F^{-1}(p) \\ &= \left(R - u + \frac{\beta}{\xi}\right) \cdot \left(\frac{\overline{F}(R)}{1-p}\right)^{\xi} - \left(-u + \frac{\beta}{\xi}\right)\end{aligned}$$

Let N be the total annual number of losses, then the ORC at the confidence level of c can be approximated through the single loss approximation as follows.

$$\text{ORC} \cong F^{-1}\left(1 - \frac{1-c}{N}\right)$$

By substituting $\overline{F}(R) \cong N_R / N$, we obtain the ORC formula described in Section 4.

$$\text{ORC} = \left(R - u + \frac{\beta}{\xi}\right) \cdot \left(\frac{1}{1-c} \cdot N_R\right)^{\xi} - \left(-u + \frac{\beta}{\xi}\right)$$

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